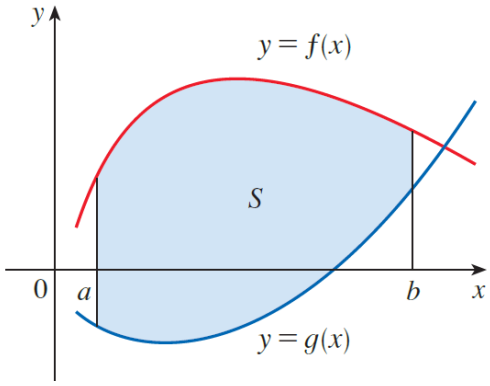
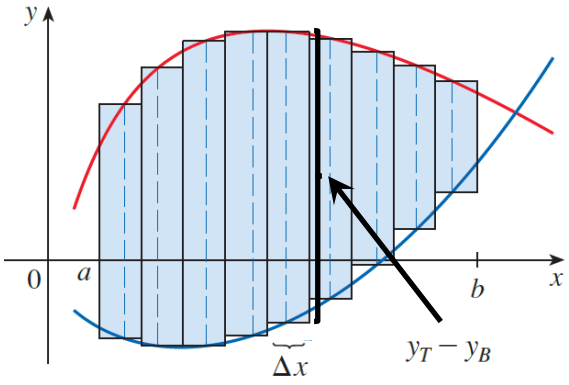
Chapter 6: Applications of Integration
6.1: Area Between Curves

Area Between Curves: Integrating With Respect to x

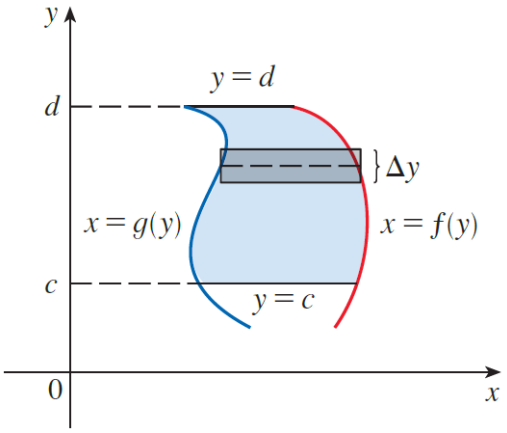


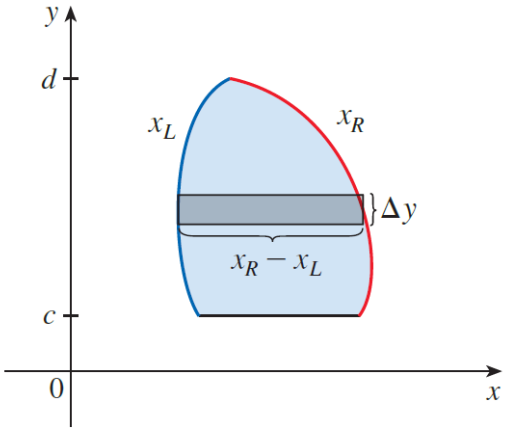


$$A = \lim_{n\to\infty} \sum_{i=1}^{n} (y_T - y_B)\,\Delta x = \int_a^b (y_T - y_B)\,dx$$

$$= \int_a^b [f(x) - g(x)]\,dx$$

Area Between Curves: Integrating With Respect to y

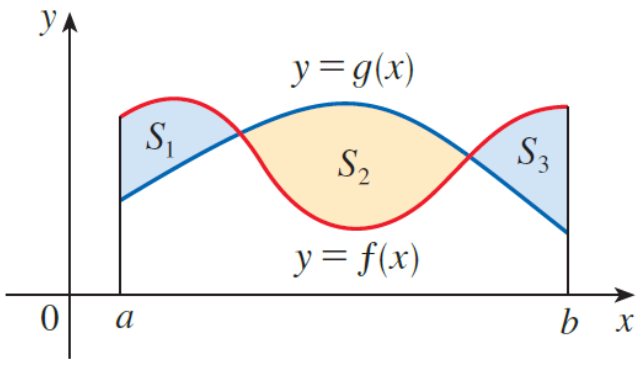




$$A = \int_c^d (x_R - x_L)\,dy$$

$$= \int_c^d [f(y) - g(y)]\,dy$$

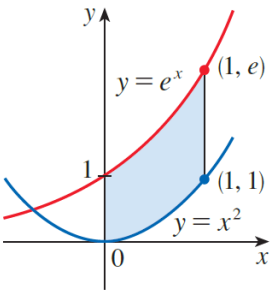
Notes:



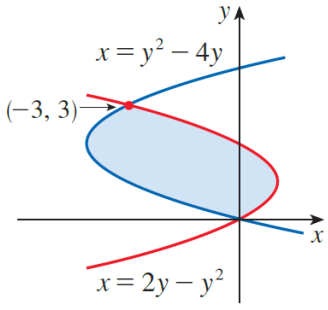
$$A = \int_a^b |f(x) - g(x)|\,dx$$

(a) Set up an integral for the area of the shaded region.
(b) Evaluate the integral to find the area.

2.

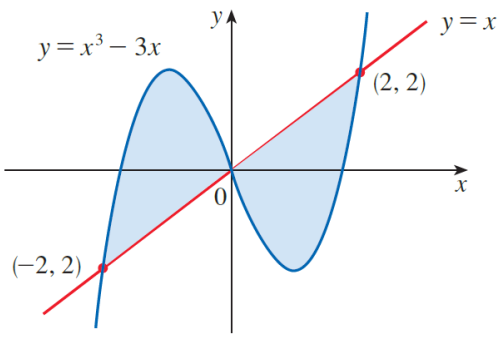


4.

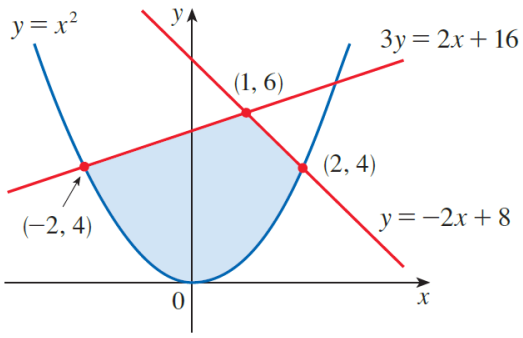


Find the area of the shaded region.

5.



6.



Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Then find the area of the region.

14. $y = \cos x, \quad y = e^x, \quad x = \pi/2$

18. $4x + y^2 = 12, \quad x = y$

Sketch the region enclosed by the given curves and find its area.

21. $x = 2y^2, \quad x = 4 + y^2$

23. $y = \sqrt[3]{2x}, \quad y = \frac{1}{2}x$

Sketch the region enclosed by the given curves and find its area.

24. $y = x^3, \quad y = x$

34. $y = 4 - 2\cosh x, \quad y = \frac{1}{2}\sinh x$

Use calculus to find the area of the triangle with the given vertices.

41. $(0, 0), \quad (3, 1), \quad (1, 2)$

Evaluate the integral and interpret it as the area of a region. Sketch the region.

43. $\int_0^{\pi/2} |\sin x - \cos 2x| \, dx$