Differential equation (DE)

Definition:

$$\frac{dy}{dx} + 5y = e^x \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Classification of DE

1. **Classification by type**

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + 6y = 0 \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Classification of DE

3. **Classification by linearity**

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y - g(x) = 0$$

$$x^3\frac{d^3y}{dx^3} + x\frac{dy}{dx} - 5y = e^x \qquad (1-y)y' + 2y = e^x \qquad \frac{d^2y}{dx^2} + \sin y = 0$$

Forms of a First-Order ODE

1. **Normal form**

$$\frac{d^n y}{dx^n} = f(x, y, y', \ldots, y^{(n-1)})$$

2. **Differential form**

$$M(x, y)\,dx + N(x, y)\,dy = 0.$$

Solutions of ODEs

➢ **Solution forms (explicit & implicit solutions)**

$$y = \phi(x) \qquad G(x, y) = 0$$

$$\frac{dy}{dx} = xy^{1/2} \Rightarrow y = \tfrac{1}{16}x^4 \qquad \frac{dy}{dx} = -\frac{x}{y} \qquad x^2 + y^2 = 25$$

Solutions of ODEs

➢ **Families of Solutions (one or n parameter solutions)**

$$F(x, y, y') = 0 \qquad F(x, y, y', \ldots, y^{(n)}) = 0$$

$$\Downarrow \qquad \Downarrow$$

$$G(x, y, c) = 0 \qquad G(x, y, c_1, c_2, \ldots, c_n) = 0$$

$$\frac{dy}{dx} = xy^{1/2} \Rightarrow y = \left(\tfrac{1}{4}x^2 + c\right)^2$$

Solutions of ODEs

- **Solutions types**

1. Particular Solutions

$$xy' - y = x^2 \sin x \qquad\qquad y = cx - x\cos x$$

2. Singular Solutions

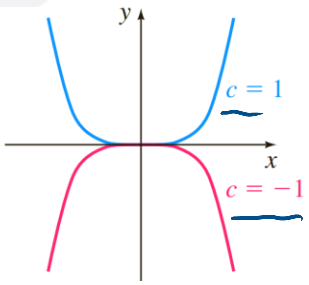
$$\frac{dy}{dx} = xy^{1/2} \Rightarrow y = \left(\tfrac{1}{4}x^2 + c\right)^2$$

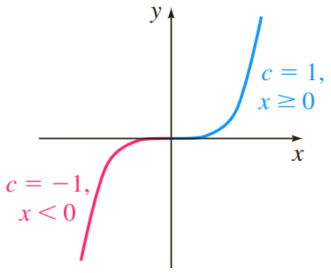
Solutions of ODEs

- **Solutions types**

3. Piecewise-Defined Solution

$$xy' - 4y = 0 \Rightarrow y = cx^4$$





$$y = \begin{cases} -x^4, & x < 0 \\ x^4, & x > 0 \end{cases}$$

System of ODEs

$$\frac{dx}{dt} = f(t, x, y)$$

$$\frac{dy}{dt} = g(t, x, y)$$

Solutions are

$$x = \phi_1(t)$$
$$y = \phi_2(t)$$

Determine whether the equation is linear or nonlinear

$$(1 - x)y'' - 4xy' + 5y = \cos x$$

$$\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r + u)$$

Verify that the indicated function is an explicit solution of the given differential equation

$$2y' + y = 0; \quad y = e^{-x/2}$$

Verify that the piecewise-defined function

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

is a solution of the differential equation $xy' - 2y = 0$