Chapter 4: Applications of Differentiation

4.4: Indeterminate Forms and l'Hospital's Rule

Indeterminate Forms

$$\frac{0}{0} \quad , \quad \frac{\infty}{\infty} \quad , \quad 0 \cdot \infty \quad , \quad \infty - \infty \quad , \quad 0^0 \quad , \quad \infty^0 \quad , \quad 1^\infty$$

L'Hospital's Rule

Suppose f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0$$

or that

$$\lim_{x \to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \to a} g(x) = \pm\infty$$

Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).

Notes:

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

9. $\lim_{x \to 4} \dfrac{x^2 - 2x - 8}{x - 4}$

11. $\lim_{x \to 1} \dfrac{x^7 - 1}{x^3 - 1}$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

13. $\lim_{x \to \pi/4} \dfrac{\sin x - \cos x}{\tan x - 1}$

14. $\lim_{x \to 0} \dfrac{\tan 3x}{\sin 2x}$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

15. $\lim_{t \to 0} \frac{e^{2t} - 1}{\sin t}$

21. $\lim_{x \to 0^+} \frac{\ln x}{x}$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

24. $\lim_{t \to 0} \frac{8^t - 5^t}{t}$

25. $\lim_{x \to 0} \frac{\sqrt{1 + 2x} - \sqrt{1 - 4x}}{x}$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

28. $\lim_{x \to 0} \dfrac{\sinh x - x}{x^3}$

33. $\lim_{x \to 0} \dfrac{x 3^x}{3^x - 1}$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

34. $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x}$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

46. $\lim_{x \to -\infty} x \ln\left(1 - \frac{1}{x}\right)$

47. $\lim_{x \to \infty} x^3 e^{-x^2}$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

48. $\lim_{x \to \infty} x^{3/2} \sin(1/x)$

49. $\lim_{x \to 1^+} \ln x \tan(\pi x/2)$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

52. $\lim_{x \to 0} (\csc x - \cot x)$

53. $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

54. $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\tan^{-1} x} \right)$

56. $\lim_{x \to \infty} (x - \ln x)$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

57. $\lim_{x \to 0^+} x^{\sqrt{x}}$

60. $\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx}$

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

64. $\lim_{x \to \infty} x^{e^{-x}}$

65. $\lim_{x \to 0^+} (4x + 1)^{\cot x}$

90. For what values of a and b is the following equation true?

$$\lim_{x \to 0} \left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2} \right) = 0$$