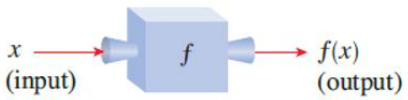
Review C: Functions

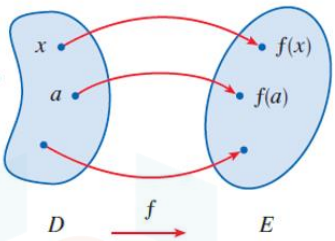
1) Functions
2) Domain & Range of Functions
3) Graphs of some Basic Functions
4) Basic Transformations of Functions
5) Composite Functions

1) Functions

A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E



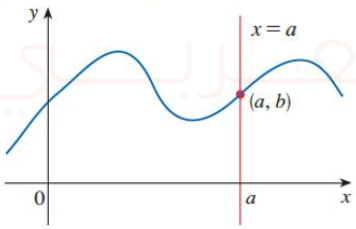
The set D is called the **domain** of the function.
The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain.



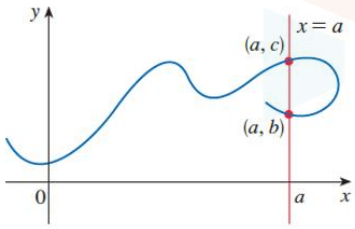
x is called an **independent variable**.
$f(x)$ is called a **dependent variable**.

The Vertical Line Test

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.



This curve represents a function.

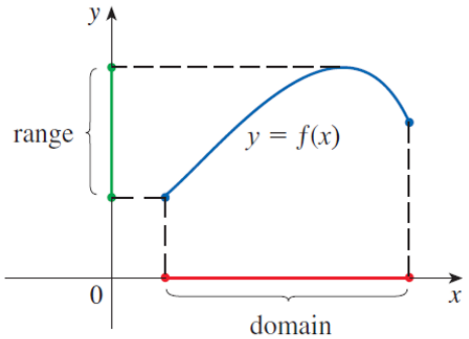


This curve doesn't represent a function.

2) Domain & Range of Functions

Domain:

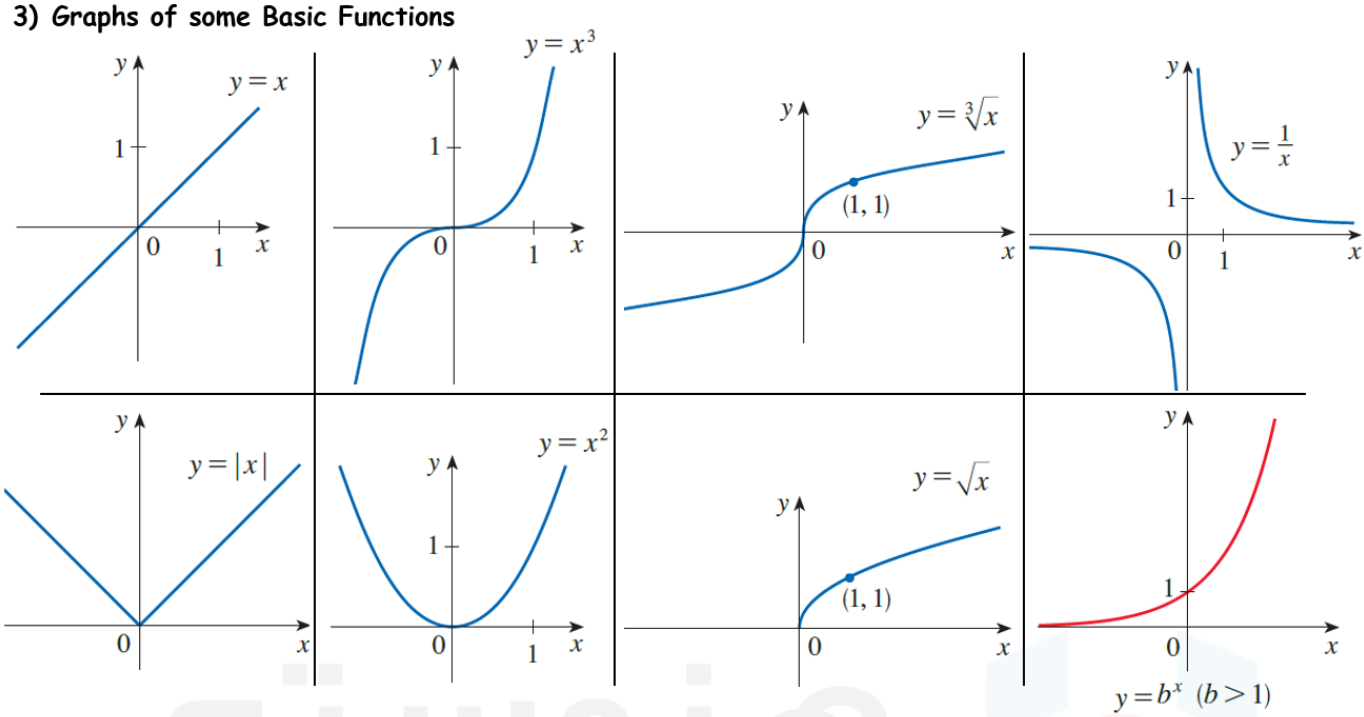
The domain is the set of all possible values of x that we can substitute in $f(x)$ such that $f(x)$ is defined.



Range:

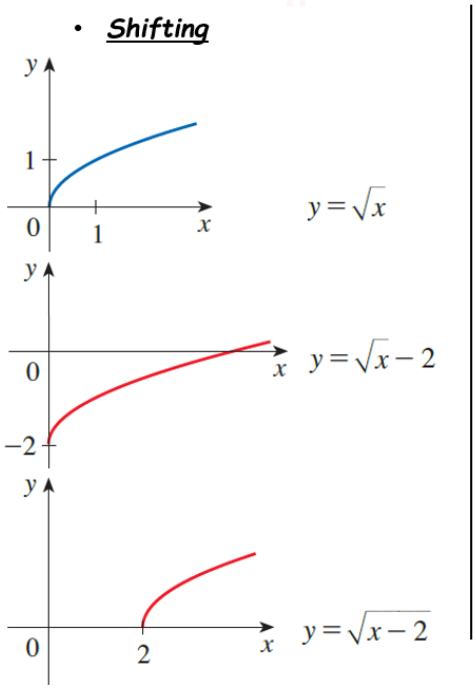
The range is the set of all possible values of y corresponding to the values of x in the domain.

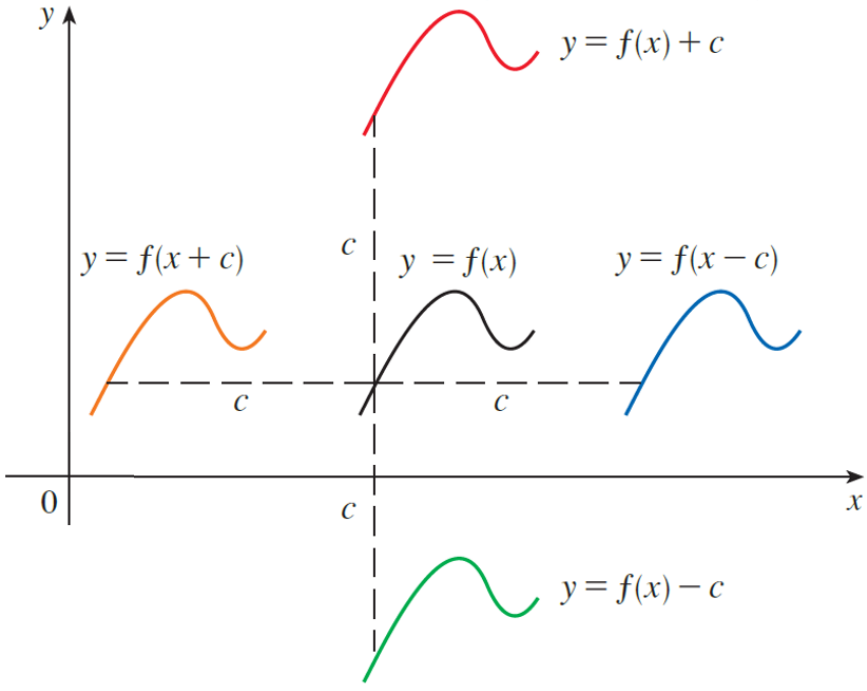
3) Graphs of some Basic Functions



4) Basic Transformations of Functions

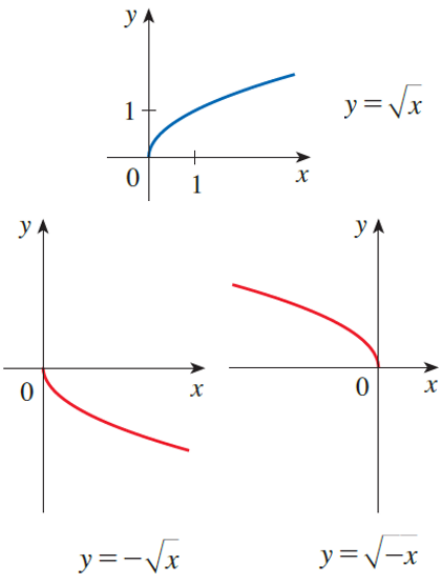
- ***Shifting***

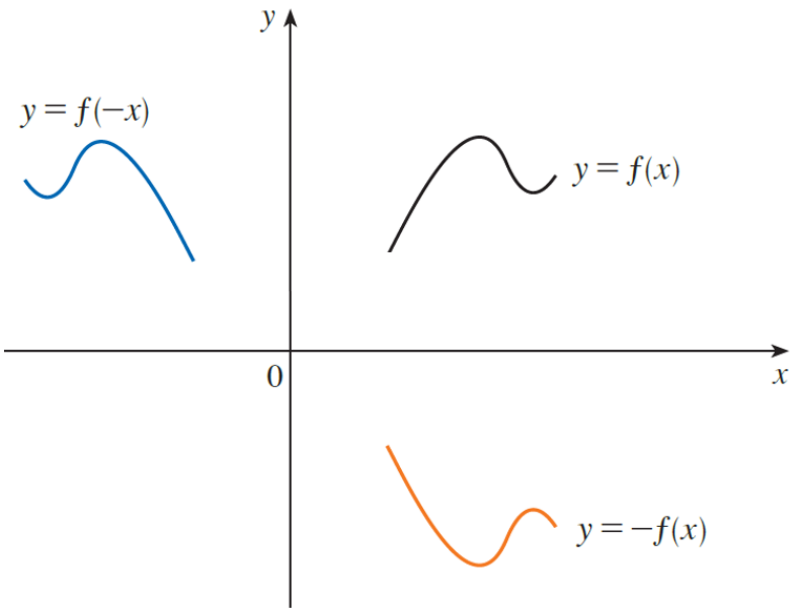




4) Basic Transformations of Functions

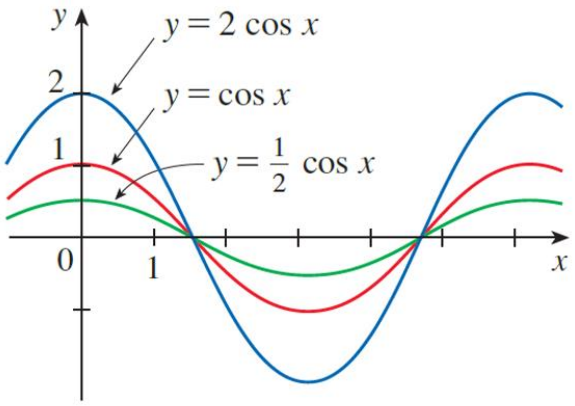
- ***Reflection***

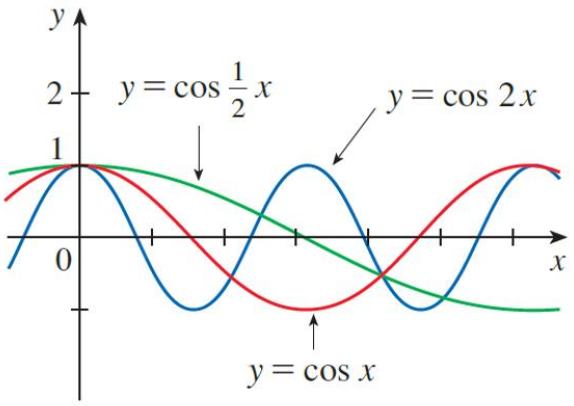




4) Basic Transformations of Functions

- ***Scaling***



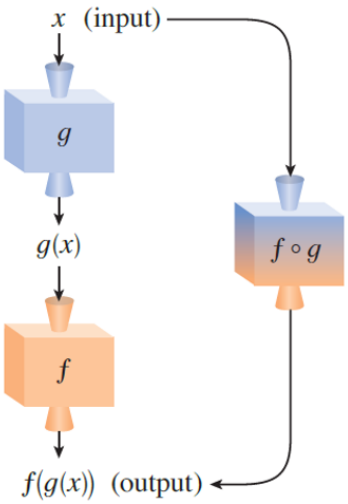


5) Composite Functions

Definition

Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$



EXAMPLE

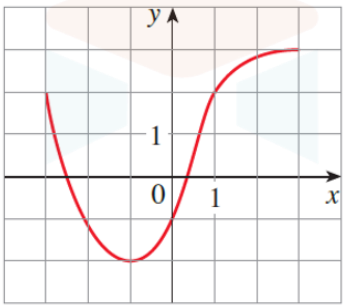
If $f(x) = x^2$ and $g(x) = x - 3$

We have

$$(f \circ g)(x) = f(g(x)) = f(x - 3) = (x - 3)^2$$

$$(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 3$$

1. The graph of a function f is given



(a) State the value of $f(-1)$.

(b) Estimate the value of $f(2)$.

(c) For what values of x is $f(x) = 2$?

(d) Estimate the values of x such that $f(x) = 0$.

(e) State the domain and range of f.

2. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.

(a) $f(x) = \dfrac{2x + 1}{x^2 + x - 2}$

(b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2 + 1}$

(c) $h(x) = \sqrt{4 - x} + \sqrt{x^2 - 1}$

4. How are graphs of the functions obtained from the graph of f?

(a) $y = -f(x)$

(b) $y = 2f(x) - 1$

(c) $y = f(x - 3) + 2$

5. Without using a calculator, make a rough sketch of the graph.

(a) $y = x^3$

(b) $y = (x + 1)^3$

(c) $y = (x - 2)^3 + 3$

(d) $y = 4 - x^2$

5. Without using a calculator, make a rough sketch of the graph.

(e) $y = \sqrt{x}$

(f) $y = 2\sqrt{x}$

(g) $y = -2^x$

(h) $y = 1 + x^{-1}$

6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

(a) Evaluate $f(-2)$ and $f(1)$.

(b) Sketch the graph of f.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.

(a) $f \circ g$

(b) $g \circ f$

(c) $g \circ g \circ g$