Solving Linear Equations

$x + 4 = 7 \quad \Rightarrow$ Equation

$x \Rightarrow unknown\ (variable)$
$values\ of\ x \Rightarrow solutions\ (roots)$
$finding\ the\ solution \Rightarrow solving\ the\ equation$

Properties of equality

- $A = B \Leftrightarrow A + C = B + C$

- $A = B \Leftrightarrow CA = CB\ (C \neq 0)$

Solving Linear Equations

A linear equation in one variable is

$$\boldsymbol{ax + b = 0}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real numbers and $\boldsymbol{x}$ is the variable.

Linear equations

$4x - 5 = 3$

$2x = \frac{1}{2}x - 7$

$x - 6 = \frac{x}{3}$

Nonlinear equations

$x^2 + 2x = 8$

$\sqrt{x} - 6x = 0$

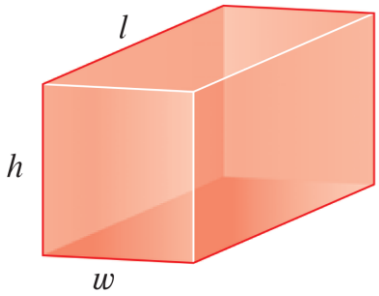
$\frac{3}{x} - 2x = 1$

Example 1: solve the equation

$$7x - 4 = 3x + 8$$

Solving for One Variable in Terms of Others

Example 2: The surface area A of the closed rectangular box can be calculated from the length l, the width w, and the height h according to the formula



$$A = 2lw + 2wh + 2lh$$

Solve for w in terms of the other variables in this equation.

Example 3: Determine whether the given value is a solution of the equation.

- $4x + 7 = 9x - 3$
  (a) $x = -2$
  (b) $x = 2$

- $\frac{1}{x} - \frac{1}{x-4} = 1$
  (a) $x = 2$,
  (b) $x = 4$

Example 4: The given equation is either linear or equivalent to a linear equation. Solve the equation.

- $2x + 3 = 7 - 3x$
- $2(1 - x) = 3(1 + 2x) + 5$
- $\frac{4}{x-1} + \frac{2}{x+1} = \frac{35}{x^2-1}$
- $(t - 4)^2 = (t + 4)^2 + 32$

Example 5: Solve the equation for the indicated variable.

- $PV = nRT$; for R
- $\frac{a+1}{b} = \frac{a-1}{b} + \frac{b+1}{a}$; for a
- $\frac{ax+b}{cx+d} = 2$; for x