Exponential Notation

If a is any real number and n is a positive integer, then the nth power of a is

$$a^n = \underbrace{a.a \dots .a}_{n \text{ factors}}$$

The number a is called the base, and n is called the exponent.

Zero and Negative Exponents

If $a \neq 0$ is a real number and n is a positive integer, then

$$a^0 = 1, \qquad a^{-n} = \frac{1}{a^n}$$

Law of Exponent

- $a^m a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

Example 1:
Eliminate negative exponents and simplify each expression.

(a) $\frac{6st^{-4}}{2s^{-2}t^{2}}$

(b) $\left(\frac{y}{3z^{3}}\right)^{-2}$

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Example 2: We can write the product 5 . 5 . 5 . 5 . 5 . 5 as .... using exponential notation.

Example 3: Evaluate each expression.

- -2^6
- $(-2)^6$
- $\left(\frac{1}{5}\right)^2 \cdot (-3)^3$

Example 4: Evaluate each expression.

- $\left(\frac{5}{3}\right)^0 \cdot 2^{-1}$
- $-2^{-3} \cdot (-2)^0$
- $\left(\frac{-2}{3}\right)^{-3}$
- $5^3 \cdot 5$
- $3^2 \cdot 3^0$
- $(2^2)^3$
- $\frac{10^7}{10^4}$

Example 5: Simplify each expression.

- x^2x^3
- $(-x^2)^3$
- $t^{-3}t^5$
- $w^{-2}w^{-4}w^5$
- $\frac{y^{10}y^0}{y^7}$
- $(2x)^2(5x^6)$
- $(2a^3a^2)^4$

Simplify each expression and eliminate any negative exponent(s).

- $(2a^2b^{-1})(3a^{-2}b^2)$

- $(9y^{-2}z^2)(3y^3z)$

- $(8x^7y^2)\left(\frac{1}{2}x^3y\right)^{-2}$

- $\frac{x^2y^{-1}}{x^{-5}}$

- $\frac{3x^{-2}y^5}{9x^{-3}y^2}$

- $\left(\frac{y^{-1}}{x^{-2}}\right)^{-1}\left(\frac{3x^{-3}}{y^2}\right)^{-2}$

- $\frac{\frac{1}{2}a^{-3}b^{-4}}{2a^{-5}b^{-1}}$

- $\left(\frac{q^{-1}r^{-1}s^{-2}}{r^{-5}sq^{-8}}\right)^{-1}$

- $\left(\frac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3}$

Scientific Notation

A positive number ***x*** is said to be written in ***scientific notation*** if it is expressed as follows:

$$x = a \times 10^n \qquad where\ 1 \le a < 10\ \ and\ \boldsymbol{n}\ is\ an\ integer$$

$3 \times 10^9 = 3000{,}000{,}000$ $\qquad$ $1.2 \times 10^{-8} = 0.000000012$

Example1:

Write each number in scientific notation.

(a) 56920

(b) 0.000093

Example 2:

Write each number in scientific notation.

- 69,300,000
- 7,200,000,000,000
- 0.000028536
- 0.0001213

Example 3:

Write each number in decimal notation.

- 3.19×10^5

- 2.670×10^{-8}

- 7.1×10^{14}

- 8.55×10^{-3}

Radicals

Definition of nth Root

If n is any positive integer, then the principal n^{th} root of a is defined as follows:

$$\sqrt[n]{a} = b, \qquad \textit{which means} \qquad b^n = a$$

If n is even, we must have $a \geq 0$ and $b \geq 0$

Properties of n^{th} Roots

- $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
- $\sqrt[n]{a^n} = a$ if n is odd
- $\sqrt[n]{a^n} = |a|$ if n is even

Example 1: Simplify the following Radicals

- $\sqrt{32}$
- $\sqrt[3]{-16x^4}$
- $-8\sqrt{12} + \sqrt{3}$
- $\sqrt[3]{8x^4} + \sqrt[3]{-x} + 4\sqrt[3]{27x}$

Example 2: Evaluate each expression

- $\sqrt{16}$
- $\sqrt[4]{16}$
- $\sqrt[3]{-64}$
- $\sqrt{\frac{27}{4}}$
- $\sqrt{7}\sqrt{28}$
- $\sqrt[3]{2}\sqrt[3]{32}$
- $\sqrt[6]{\frac{1}{2}}\sqrt[6]{128}$

Example 3: Simplify each expression. Assume that all variables are positive when they appear

- $\sqrt[3]{27}$
- $\sqrt[3]{-8}$
- $\sqrt{700}$
- $\sqrt[3]{-8x^4}$
- $\sqrt[4]{x^{12}y^8}$
- $\sqrt[3]{\sqrt{64x^6}}$

Example 4: Simplify each expression. Assume that all variables are positive when they appear

- $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}}$
- $\sqrt{15x^2}\sqrt{5x}$
- $(\sqrt[3]{3}\sqrt{10})^4$
- $(5\sqrt{8})(-3\sqrt{3})$
- $6\sqrt{5} - 4\sqrt{5}$

Example 5: Simplify the expression. Assume that all letters denote positive numbers

- $\sqrt{32} + \sqrt{18}$
- $\sqrt[3]{54} - \sqrt[3]{16}$
- $\sqrt{9a^3} - \sqrt{a}$
- $\sqrt[3]{2y^4} - \sqrt[3]{2y}$
- $\sqrt{81x^2 + 81}$

Rational Exponent

Definition of Rational Exponent

If m and n are integers and $n > 0$, then

$$a^{m/n} = (\sqrt[n]{a})^m \qquad \text{or} \qquad a^{m/n} = \sqrt[n]{a^m}$$

If n is even, then we require that $a \geq 0$.

Example 1:
Use the Laws of Exponents with Rational Exponents to simplify the following:

- $a^{1/3}a^{7/3}$
- $\dfrac{a^{2/5}a^{7/5}}{a^{3/5}}$

Example 2: complete the following table

Radical expression	**Exponential expression**
$\dfrac{1}{\sqrt{3}}$	
$\sqrt[3]{7^2}$	
	$4^{2/3}$
	$10^{-3/2}$
$\sqrt[5]{5^3}$	
	$2^{-1.5}$
	$a^{2/5}$

Example 3: Simplify the expression and eliminate any negative exponent(s). Assume that all letters denote positive numbers.

- $x^{3/4}x^{5/4}$
- $(u^4v^6)^{-1/3}$
- $\dfrac{(2y^{4/3})^2 y^{-2/3}}{y^{7/3}}$
- $\left(\dfrac{x^{-2/3}}{y^{1/2}}\right)\left(\dfrac{x^{-2}}{y^{-3}}\right)^{1/6}$
- $(8a^6b^{3/2})^{2/3}$
- $\left(\dfrac{x^8y^{-4}}{16y^{4/3}}\right)^{-1/4}$

Rationalizing the Denominator; Standard Form

$$\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{a}}\cdot 1=\frac{1}{\sqrt{a}}\cdot\frac{\sqrt{a}}{\sqrt{a}}=\frac{\sqrt{a}}{a}$$

$$\sqrt[n]{a^m}\sqrt[n]{a^{n-m}}=\sqrt[n]{a^{m+n-m}}=\sqrt[n]{a^n}=a$$

Note: standard form = denominator with no radicals

Example 1:

Put each fractional expression into standard form by rationalizing the denominator.

(a) $\frac{2}{\sqrt{3}}$

(b) $\frac{1}{\sqrt[3]{5}}$

(c) $\sqrt[7]{\frac{1}{a^2}}$

Notes:

If a Denominator Contains the Factor	Multiply by	To Obtain a Denominator Free of Radicals
$\sqrt{3}$	$\sqrt{3}$	$(\sqrt{3})^2=3$
$\sqrt{3}+1$	$\sqrt{3}-1$	$(\sqrt{3})^2-1^2=3-1=2$
$\sqrt{2}-3$	$\sqrt{2}+3$	$(\sqrt{2})^2-3^2=2-9=-7$
$\sqrt{5}-\sqrt{3}$	$\sqrt{5}+\sqrt{3}$	$(\sqrt{5})^2-(\sqrt{3})^2=5-3=2$
$\sqrt[3]{4}$	$\sqrt[3]{2}$	$\sqrt[3]{4}\cdot\sqrt[3]{2}=\sqrt[3]{8}=2$

Example 2: Rationalize the denominator of each expression. Assume that all variables are positive when they appear.

- $\frac{1}{\sqrt{2}}$
- $\frac{\sqrt{3}}{5-\sqrt{2}}$
- $\frac{-\sqrt{3}}{\sqrt{5}}$
- $\frac{2-\sqrt{5}}{2+3\sqrt{5}}$
- $\frac{9}{\sqrt[4]{2}}$
- $\sqrt{\frac{x}{5}}$
- $\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$
- $\frac{1}{\sqrt[4]{x^3}}$

Rationalizing Numerators

Example 1:
Rationalize the numerator of the following expression.

$$\frac{\sqrt{x+h}-\sqrt{x}}{h} \qquad (h \neq 0)$$