Solving Quadratic Equations

Quadratic Equations

A quadratic equation is

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers with $a \neq 0$.

Zero – Product Property

$$(A)(B) = 0 \text{ if and only if } A = 0 \text{ or } B = 0$$

Solving a Quadratic Equation by Factoring

Example 1: Find all real solutions of the equation $x^2 + 5x = 24$.

Solving a simple quadratic Equation

The solutions of $x^2 = c \Rightarrow x = \sqrt{c}$ and $x = -\sqrt{c}$.

Example 2: Find all real solutions of each equation.

(a) $x^2 = 5$

(b) $(x - 4)^2 = 5$

Completing the Square

To make $x^2 + bx$ a perfect square, add $\left(\frac{b}{2}\right)^2$, (**the square of half the coefficient of x**). This gives the perfect square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example 3: Find all real solutions of each equation.

(a) $x^2 - 8x + 13 = 0$

(b) $3x^2 - 12x + 6 = 0$

The quadratic formula

The roots of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 4: Find all real solutions of the equation.

$$4x^2 + 12x + 9 = 0$$

The Discriminant

The **discriminant** of the general quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$ is $D = b^2 - 4ac$.

- If $D > 0$, then the equation has two distinct real solutions.
- If $D = 0$, then the equation has exactly one real solution.
- If $D < 0$, then the equation has no real solution.

Example 5: Use the discriminant to determine how many real solutions each equation has.

(a) $x^2 + 4x - 1 = 0$

(b) $4x^2 - 12x + 9 = 0$

(c) $\frac{1}{3}x^2 - 2x + 4 = 0$

Example 6: Find all real solutions of the equation by factoring

- $x^2 + x - 12 = 0$
- $2y^2 + 7y + 3 = 0$
- $(2x - 5)^2 = 81$

Example 7: Find all real solutions of the equation by completing the square.

- $x^2 + 2x - 5 = 0$
- $3x^2 - 6x - 1 = 0$

Example 8: Find all real solutions of the quadratic equation using quadratic formula.

- $x^2 - 2x - 15 = 0$
- $9x^2 + 12x + 4 = 0$

Example 9: Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

- $x^2 - 6x + 1 = 0$
- $4x^2 + 5x + \frac{13}{8} = 0$