Algebraic Expression

A **Variable** is a letter that can represent any number from a given set of numbers.

Algebraic expression.

$$2x^2 - 3x + 4 \qquad \sqrt{x} + 10 \qquad \frac{y - 2z}{y^2 + 4}$$

A **monomial** is an expression of the form ax^k , where a is a real number and k is a nonnegative integer.

A **binomial** is a sum of two monomials

A **trinomial** is a sum of three monomials.

In general, a sum of monomials is called a **polynomial.**

polynomial

A **polynomial** is an expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_0, a_1, \ldots, a_n$ are real numbers, and n is a nonnegative integer. If $a_n \neq 0$, then the polynomial has **degree n**. The **monomials** $a_k x^k$ that make up the polynomial are called the **terms** of the polynomial

Polynomial	Type	Terms	Degree
$2x^2 - 3x + 4$	trinomial	$2x^2, -3x, 4$	2
$x^8 + 5x$	binomial	$x^8, 5x$	8
$8 - \frac{1}{2}x^3 + x^2$	four terms	$-\frac{1}{2}x^3, x^2, -x, 8$	3
$5x + 1$	binomial	$5x, 1$	1
$9x^5$	monomial	$9x^5$	5
6	monomial	6	0

Example 1: Which of the following expressions are polynomials?

- $2x^3 - \frac{1}{2}x + \sqrt{3}$

- $x^2 - \frac{1}{2} - 3\sqrt{x}$

- $\frac{1}{x^2+4x+7}$

- $\sqrt[3]{8x^6 - 5x^3 + 7x - 3}$

Example 2: Complete the following table

Polynomial	Type	Terms	Degree
$-2x^2 + 5x - 3$			
-8			
$\frac{1}{2}x^7$			
$x - x^2 + x^3 - x^4$			
$\sqrt{2}x - \sqrt{3}$			

Adding and Subtracting Polynomials

Distributive Property

$$\boldsymbol{a(b+c) = ab + ac}$$

Example 1:

Find the sum of the polynomials:

$$8x^3 - 2x^2 + 6x - 2 \text{ and } 3x^4 - 2x^3 + x^2 + x$$

Subtracting Polynomials

Example 2:

Find the difference:

$$(3x^4 - 4x^3 + 6x^2 - 1) - (2x^4 - 8x^2 - 6x + 5)$$

Example 3: Find the sum, difference, or product.

- $(6x - 3) + (3x + 7)$
- $(5x^3 + 4x^2 - 3x) - (x^2 + 7x + 2)$
- $(3 - 7x) - (11 + 4x)$
- $3(x - 1) + 4(x + 2)$
- $(-2x^2 - 3x + 1) + (3x^2 + 5x - 4)$
- $8(2x + 5) - 7(x - 9)$
- $(x^2 - x + 2) + (2x^2 - 3x + 5) - (x^2 + 1)$
- $3y(2y + 5)$
- $8(1 - y^3) + 4(1 + y + y^2 + y^3)$
- $v^3(v - 9) - 2v^2(2 - 2v)$

Multiplying Polynomials

Example 1:

Find the product: $(2x+5)(x^2-x+2)$

FOIL (First, Outer, Inner, Last): method of multiplying two binomials.

Outer

First

$(ax+b)(cx+d)$

inner

Last

First Outer Inner Last

$= ax\cdot cx + ax\cdot d + b\cdot cx + b\cdot d$

$= acx^2 + adx + bcx + bd$

$= acx^2 + (ad+bc)x + bd$

Example 2:

Multiply the Binomial using FOIL :$(2x+1)(3x-5)$

Example 3: Multiply the following polynomials

- $(x+2)\left(x^2+2x+3\right)$

- $(1+2x)(x^2-3x+1)$

- $(x+1)(x^2+2x-4)$

- $(2x-3)(x^2+x+1)$

Example 4: Multiply the polynomials using the FOIL method.

- $(x+2)(x+4)$

- $(x+4)(x-2)$

- $(x-3y)(-2x+y)$

- $(2r - 5s)(3r - 2s)$

Special Product Formulas

If A and B are any real numbers or algebraic expressions, then

- $(A+B)(A-B)=A^2-B^2$ Sum and difference of same terms
- $(A+B)^2=A^2+2AB+B^2$ Square of a sum
- $(A-B)^2=A^2-2AB+B^2$ Square of a difference
- $(A+B)^3=A^3+3A^2B+3AB^2+B^3$ Cube of a sum
- $(A-B)^3=A^3-3A^2B+3AB^2-B^3$ Cube of a difference
- $(x-a)(x^2+ax+a^2)=x^3-a^3$ Difference of Two Cubes
- $(x+a)(x^2-ax+a^2)=x^3+a^3$ Sum of Two Cubes

Example 1:

Use the Special Product Formulas to find each product.

$(x^2-2)^3$

$(x+y-1)(x+y+1)$

Example 2: Multiply the algebraic expressions using a Special Product Formula

- $(5x+1)^2$
- $(x+6)(x-6)$
- $(2-7y)^2$
- $(5-y)(5+y)$
- $(2+y^3)^2$
- $(x+3y)(x-3y)$
- $(2u+v)(2u-v)$
- $\left(1+x^{2/3}\right)\left(1-x^{2/3}\right)$
- $(\sqrt{x}+2)(\sqrt{x}-2)$
- $(1-b)^2(1+b)^2$
- $(x-3)^3$
- $\left(x+(2+x^2)\right)\left(x-(2+x^2)\right)$

Factoring

Factoring →

$$x^2 - 16 = (x - 4)(x + 4)$$

← **Expanding**

We say that $x - 4$ and $x + 4$ are **factors** of $x^2 - 16$.

Common Factors

Example 1: Factor each expression.

(a) $3x^2 - 6x$

(b) $8x^4y^2 + 6x^3y^3 - 2xy^4$

Factoring Trinomials

To factor a trinomial of the form $x^2 + Bx + C$, we note that

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$\Rightarrow \quad a + b = B \qquad \Rightarrow ab = C.$$

Example 2:

Factor: $x^2 + 7x + 10$

Factoring Trinomial by Grouping

To factor a trinomial of the form $Ax^2 + Bx + C$ with A$\neq$ 1:

1. Find two numbers a and b such that $ab = AC$ and $a + b = B$
2. Rewrite $\Rightarrow Ax^2 + Bx + C = Ax^2 + ax + bx + C$.
3. Factor $Ax^2 + ax + bx + C$ by grouping each two terms with the same common factor

Example 3:

Factor: $2x^2 + 5x + 3$

Prime Polynomial

A prime polynomial has the form $x^2 + a^2$, where a is real number

Example 4:

Show that $x^2 + 9$ is prime

Example 5: Factor out the common factor.

- $3x + 6$
- $-2x^3 + x$
- $3x^2y - 6xy^2 + 12xy$
- $(z+2)^2 - 5(z+2)$

Example 6: Factor the trinomial.

- $x^2 + 8x + 7$
- $2x^2 - 5x - 7$
- $x^2 - 7x - 8$
- $(3x+2)^2 + 8(3x+2) + 12$

Example 7: Factor each polynomial completely. If the polynomial cannot be factored, say it is prime.

- $x^2 + 11x + 10$
- $3y^3 - 18y^2 - 48y$
- $x^2 + x + 1$
- $5(3x - 7) + x(3x - 7)$

Special Factoring Formulas

- $A^2 - B^2 = (A - B)(A + B)$ Difference of squares
- $A^2 + 2AB + B^2 = (A + B)^2$ Perfect square
- $A^2 - 2AB + B^2 = (A - B)^2$ Perfect square
- $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$ Difference of cubes
- $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ Sum of cubes

Example 1:
Factor each expression.
(a) $4x^2 - 25$

(b) $x^2 + 6x + 9$

Factoring an Expression Completely

Example 2:

Factor each expression completely.

(a) $2x^4 - 8x^2$

(b) $x^5y^2 - xy^6$

Factoring Expressions with Fractional Exponents

Example 3:

Factor each expression.

(a) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$

(b) $(2+x)^{-2/3}x + (2+x)^{1/3}$

Factoring by Grouping Terms

Example 4:

Factor each polynomial.

(a) $x^3 + x^2 + 4x + 4$

(b) $x^3 - 2x^2 - 9x + 18$

Example 5: Factor the difference of squares

- $x^2 - 25$
- $36x^2 - 9$
- $16y^2 - z^2$
- $x^2 - (y+5)^2$

Example 6: Factor the perfect square

- $x^2 + 2x + 1$
- $16a^2 + 24a + 9$
- $z^2 - 12z + 36$
- $9u^2 - 6uv + v^2$

Example 7: Factor the sum or difference of two cubes

- $x^3 - 27$

- $y^3 - 64$

- $8x^3 + 27$

Example 8: Factor the expression by grouping terms.

- $x^3 + 4x^2 + x + 4$

- $5x^2 - 15x + x - 3$

- $6x^2 + 21x + 8x + 28$

- $18x^3 + 9x^2 + 2x + 1$

Example 9: Factor the expression completely. Begin by factoring out the lowest power of each common factor.

- $x^{5/2} - x^{1/2}$

- $x^{-3/2} + 2x^{-1/2} + x^{1/2}$

- $(x-1)^{7/2} - (x-1)^{3/2}$

Example 10: Factor each polynomial completely.

- $x^6 - 2x^3 + 1$

- $x^4 + x^3 + x + 1$

- $(x-1)(x+2)^2 - (x-1)^2(x+2)$

- $(x+1)^3 x - 2(x+1)^2 x^2 + x^3(x+1)$

Completing the square

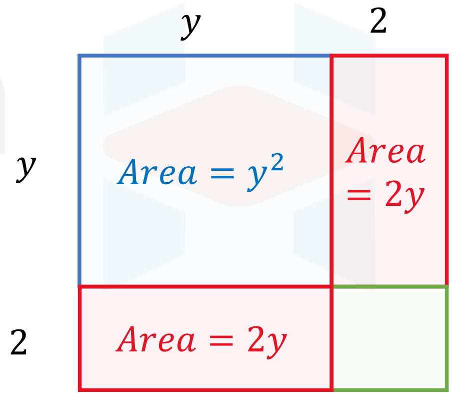
$x^2 + 2ax + a^2 = (x + a)^2$

$x^2 - 2ax + a^2 = (x - a)^2$

To complete the Square of $x^2 + bx$

- Multiply b by $\frac{1}{2}$ and then square the result $\Rightarrow \left(\frac{1}{2}b\right)^2$.
- Add $\left(\frac{1}{2}b\right)^2$ to $x^2 + bx$ to get $x^2 + bx + \left(\frac{1}{2}b\right)^2 = \left(x + \frac{b}{2}\right)^2$

Example 1:
Determine the number that must be added to the expression to complete the square. $y^2 + 4y$



Example 2: Determine what number should be added to complete the square of each expression. Then factor each expression.

- $x^2 + 10x$

- $y^2 - 6y$