

Simplifying Rational Expressions

$$\frac{x^3 - x}{x^2 - 5x + 6} \Rightarrow \text{Rational Expression}, \quad \frac{x}{\sqrt{x^2 + 1}} \Rightarrow \text{Not rational (Fractional)}$$

Cancellation Property

$$\frac{ac}{bc} = \frac{a}{b} \quad (\text{if } b \neq 0, c \neq 0)$$

Example 1: Simplify

$$\frac{x^2 - 1}{x^2 + x - 2}$$

Example 2: Which of the following are rational expressions?

(a) $\frac{3x}{x^2 - 1}$

(b) $\frac{\sqrt{x+1}}{2x+3}$

(c) $\frac{x(x^2 - 1)}{x+3}$

Example 3: Reduce each rational expression to lowest terms.

• $\frac{3x+9}{x^2-9}$

• $\frac{5(x-3)(2x+1)}{10(x-3)^2}$



- $\frac{3y^2 - y - 2}{3y^2 + 5y + 2}$

- $\frac{1 - x^2}{x^3 - 1}$

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Multiplying and Dividing Rational Expressions

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational expressions, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{if } b \neq 0, d \neq 0$$
$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \quad \text{if } b \neq 0, c \neq 0, d \neq 0$$

Example 1: Perform the indicated multiplication and simplify:

$$\frac{x^2 + 2x - 3}{x^2 + 8x + 16} \cdot \frac{3x + 12}{x - 1}$$

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Example 2: Perform the indicated division and simplify:

$$\frac{x - 4}{x^2 - 4} \div \frac{x^2 - 3x - 4}{x^2 + 5x + 6}$$





Example 3: Perform the multiplication or division and simplify.

- $\frac{3x+6}{5x^2} \cdot \frac{x}{x^2-4}$

- $\frac{x^2+2x-3}{x^2-2x-3} \cdot \frac{3-x}{3+x}$

- $\frac{t-3}{t^2+9} \cdot \frac{t+3}{t^2-9}$

- $\frac{2x+1}{2x^2+x-15} \div \frac{6x^2-x-2}{x+3}$

- $\frac{\frac{5x^2-7x-6}{2x^2+3x-5}}{\frac{15x^2+14x+3}{2x^2+13x+20}}$

- $\frac{x^2+x-6}{x^2+4x-5} \cdot \frac{x^2-25}{x^2+2x-15}$



Adding and Subtracting Rational Expressions with Equal Denominators

If $\frac{a}{b}$ and $\frac{c}{b}$ are two rational expressions, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \text{ if } b \neq 0$$

Example 1: Perform the indicated operations and simplify.

$$\frac{2x^2 - 4}{2x + 5} + \frac{x + 3}{2x + 5} \quad x \neq -\frac{5}{2}$$

Example 2: Perform the indicated operations and simplify.

$$\frac{2x}{x - 3} + \frac{5}{3 - x} \quad x \neq 3$$





Adding and Subtracting Rational Expressions with Unequal Denominators

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational expressions, then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \text{if } b \neq 0, d \neq 0$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \quad \text{if } b \neq 0, d \neq 0$$

Example 3: Perform the indicated operations and simplify.

$$\frac{3}{x-1} + \frac{x}{x+2}$$

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Example 4: Choose the statement that is not true. Assume $b \neq 0$, $c \neq 0$, and $d \neq 0$ as necessary.

(a) $\frac{ac}{bc} = \frac{a}{b}$

(b) $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

(c) $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$

(d) $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ac}{bd}$



Example 5: Perform the addition or subtraction and simplify

- $\frac{x}{2} + \frac{5}{2}$

- $\frac{x}{x-4} - \frac{3}{x+6}$

- $1 + \frac{1}{x+3}$

- $\frac{1}{x^2} + \frac{1}{x^2+x}$

- $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

- $\frac{2}{x} + \frac{3}{x-1} - \frac{4}{x^2-x}$



- $\frac{1}{x^2+3x+2} - \frac{1}{x^2-2x-3}$

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Compound (Complex) Fractions

Compound fraction:

Numerator or denominator, or both are fractional expressions

Example 1: Simplify:

$$\frac{\frac{x}{y} + 1}{1 - \frac{y}{x}}$$

Example 2: Simplify the compound fractional expression.

- $\frac{1 + \frac{1}{x}}{\frac{1}{x} - 2}$

- $\frac{\frac{x-3}{x-4} \frac{x+2}{x+1}}{x+3}$



- $x - \frac{y}{\frac{x+y}{y+x}}$

- $\frac{x^{-1}+y^{-1}}{(x+y)^{-1}}$

- $\frac{(1-x^2)^{1/2} + x^2(1-x^2)^{-1/2}}{1-x^2}$

- $\frac{4(x+2)^{-1}-3}{3(x+2)^{-1}-1}$





Avoiding Common Errors

| Correct multiplication property | Common error with addition |
|---|---|
| $(a \cdot b)^2 = a^2 \cdot b^2$ | $(a + b)^2 = a^2 + b^2$ |
| $\sqrt{a \cdot b} = \sqrt{a}\sqrt{b} \ (a, b \geq 0)$ | $\sqrt{a + b} = \sqrt{a} + \sqrt{b}$ |
| $\sqrt{a^2 \cdot b^2} = a \cdot b \ (a, b \geq 0)$ | $\sqrt{a^2 + b^2} = a + b$ |
| $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$ | $\frac{1}{a} + \frac{1}{b} = \frac{1}{a + b}$ |
| $\frac{ab}{a} = b$ | $\frac{a + b}{a} = b$ |
| $a^{-1} \cdot b^{-1} = (a \cdot b)^{-1}$ | $a^{-1} + b^{-1} = (a + b)^{-1}$ |

