

## Solving Linear Equations

$$x + 4 = 7 \quad \Rightarrow \text{Equation}$$

$x \Rightarrow$  unknown (variable)

values of  $x \Rightarrow$  solutions (roots)

finding the solution  $\Rightarrow$  solving the equation

## Properties of equality

- $A = B \Leftrightarrow A + C = B + C$
- $A = B \Leftrightarrow CA = CB \ (C \neq 0)$

## Solving Linear Equations

A linear equation in one variable is

$$ax + b = 0$$

where  $a$  and  $b$  are real numbers and  $x$  is the variable.

## Linear equations

$$4x - 5 = 3$$

$$2x = \frac{1}{2}x - 7$$

$$x - 6 = \frac{x}{3}$$

## Nonlinear equations

$$x^2 + 2x = 8$$

$$\sqrt{x} - 6x = 0$$

$$\frac{3}{x} - 2x = 1$$

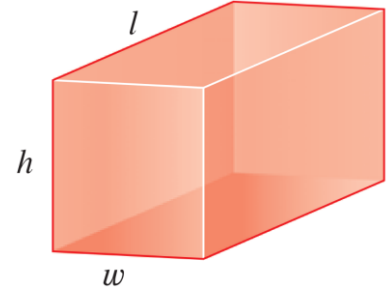
**Example 1:** solve the equation

$$7x - 4 = 3x + 8$$



### Solving for One Variable in Terms of Others

**Example 2:** The surface area  $A$  of the closed rectangular box can be calculated from the length  $l$ , the width  $w$ , and the height  $h$  according to the formula



$$A = 2lw + 2wh + 2lh$$

Solve for  $w$  in terms of the other variables in this equation.

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**Example 3:** Determine whether the given value is a solution of the equation.

- $4x + 7 = 9x - 3$   
(a)  $x = -2$   
(b)  $x = 2$

- $\frac{1}{x} - \frac{1}{x-4} = 1$   
(a)  $x = 2,$   
(b)  $x = 4$



**Example 4:** The given equation is either linear or equivalent to a linear equation. Solve the equation.

- $2x + 3 = 7 - 3x$
- $2(1 - x) = 3(1 + 2x) + 5$
- $\frac{4}{x-1} + \frac{2}{x+1} = \frac{35}{x^2-1}$
- $(t - 4)^2 = (t + 4)^2 + 32$

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**Example 5:** Solve the equation for the indicated variable.

- $PV = nRT$ ; for  $R$
- $\frac{a+1}{b} = \frac{a-1}{b} + \frac{b+1}{a}$ ; for  $a$
- $\frac{ax+b}{cx+d} = 2$ ; for  $x$



### Solving Quadratic Equations

#### Quadratic Equations

A quadratic equation is

$$ax^2 + bx + c = 0$$

where  $a, b$ , and  $c$  are real numbers with  $a \neq 0$ .

#### Zero – Product Property

$$(A)(B) = 0 \text{ if and only if } A = 0 \text{ or } B = 0$$

#### Solving a Quadratic Equation by Factoring

**Example 1:** Find all real solutions of the equation  $x^2 + 5x = 24$ .

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#### Solving a simple quadratic Equation

The solutions of  $x^2 = c \Rightarrow x = \sqrt{c}$  and  $x = -\sqrt{c}$ .

**Example 2:** Find all real solutions of each equation.

(a)  $x^2 = 5$

(b)  $(x - 4)^2 = 5$



## Completing the Square

To make  $x^2 + bx$  a perfect square, add  $\left(\frac{b}{2}\right)^2$ , (the square of half the coefficient of  $x$ ). This gives the perfect square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

**Example 3:** Find all real solutions of each equation.

(a)  $x^2 - 8x + 13 = 0$

(b)  $3x^2 - 12x + 6 = 0$

## The quadratic formula

The roots of the quadratic equation  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Example 4:** Find all real solutions of the equation.

$$4x^2 + 12x + 9 = 0$$



### The Discriminant

The **discriminant** of the general quadratic equation  $ax^2 + bx + c = 0 (a \neq 0)$  is  $D = b^2 - 4ac$ .

- If  $D > 0$ , then the equation has two distinct real solutions.
- If  $D = 0$ , then the equation has exactly one real solution.
- If  $D < 0$ , then the equation has no real solution.

**Example 5:** Use the discriminant to determine how many real solutions each equation has.

(a)  $x^2 + 4x - 1 = 0$

(b)  $4x^2 - 12x + 9 = 0$

(c)  $\frac{1}{3}x^2 - 2x + 4 = 0$

**Example 6:** Find all real solutions of the equation by factoring

•  $x^2 + x - 12 = 0$

•  $2y^2 + 7y + 3 = 0$

•  $(2x - 5)^2 = 81$



**Example 7:** Find all real solutions of the equation by completing the square.

- $x^2 + 2x - 5 = 0$

- $3x^2 - 6x - 1 = 0$

**Example 8:** Find all real solutions of the quadratic equation using quadratic formula.

- $x^2 - 2x - 15 = 0$

- $9x^2 + 12x + 4 = 0$

**Example 9:** Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

- $x^2 - 6x + 1 = 0$

- $4x^2 + 5x + \frac{13}{8} = 0$



### Solving other Types of Equations

#### An Equation Involving Fractional Expressions

**Example 1:** Solve the equation

$$\frac{3}{x} - \frac{2}{x-3} = \frac{-12}{x^2-9}$$

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#### An Equation Involving a Radical

**Example 2:** Solve the equation

$$2x = 1 - \sqrt{2-x}$$



### A Fourth-Degree Equation of Quadratic Type

**Example 3:** Find all solutions of the equation

$$x^4 - 8x^2 + 8 = 0$$

### An Equation Involving Fractional Powers

**Example 4:** Find all solutions of the equation

$$x^{1/3} + x^{1/6} - 2 = 0$$



### An Absolute Value Equation

$|X| = C$  is equivalent to  $X = C$  or  $X = -C$

**Example 5:** Solve the equation

$$|2x - 5| = 3$$



**Example 6:** Find all real solutions of the equation

- $\frac{x^2}{x+100} = 50$

- $\sqrt{2x-1} = \sqrt{3x-5}$

- $\frac{x}{2x+7} - \frac{x+1}{x+3} = 1$

- $x^4 - 5x^2 + 4 = 0$

- $|3x + 5| = 1$

- $|x - 6| = -1$



- $\sqrt{x} - 3\sqrt[4]{x} - 4 = 0$

- $4(x + 1)^{\frac{1}{2}} - 5(x + 1)^{\frac{3}{2}} + (x + 1)^{\frac{5}{2}} = 0$

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