Solving Inequalities

$$2x + 4 \leq 10$$

x	$2x + 4 \leq 10$	
1	$6 \leq 10$	✓
2	$8 \leq 10$	✓
3	$10 \leq 10$	✓
4	$12 \leq 10$	×
5	$14 \leq 10$	×

Equation: $2x + 4 = 10$, **solution:** $x = 3$

Inequality: $2x + 4 \leq 10$, **solution:** $x \leq 3$

Rules for inequalities

- $A \leq B \quad \Leftrightarrow \quad A + C \leq B + C$
- $A \leq B \quad \Leftrightarrow \quad A - C \leq B - C$
- If $C > 0$, then $A \leq B \quad \Leftrightarrow \quad CA \leq CB$
- If $C < 0$, then $A \leq B \quad \Leftrightarrow \quad CA \geq CB$
- If $A > 0$ and $B > 0$, then $A \leq B \quad \Leftrightarrow \quad \frac{1}{A} \geq \frac{1}{B}$
- If $A \leq B$ and $C \leq D$, then $A + C \leq B + D$

Example 1: Solve inequality and sketch the solution set.

$$3x - 1 \geq 3 + x$$

Example 2: Solve the inequality.

$$5 \leq 3x - 4 \leq 14$$

Solving Nonlinear Inequalities

The sign of a product or quotient

- Even number of negative terms = positive.
- Odd number of negative terms = negative.

Guidelines for Solving Nonlinear Inequalities

1. Move All Terms to One Side.
2. Find the Intervals. ( )( )
3. Make a Table or Diagram.
4. Solve.

Example 3: Solve the nonlinear inequality

$$x^2 \leq x + 2$$

Example 4: Solve the nonlinear inequality

$$(x-2)^2(x-3)(x+1) \leq 0$$

Example 5: Solve the nonlinear inequality

$$\frac{3+x}{3-x} \geq 1$$

Absolute Value Equations and Inequalities

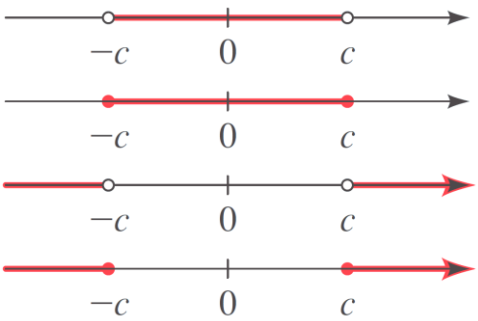
Absolute Value Equations

$|u| = a$ means $u = a$ or $u = -a$

Remember: Solve the absolute value equation

$$|1 - 2z| + 6 = 9$$

Properties of Absolute Value Inequalities

inequality	Equivalent form	Graph
• $\|x\| < c$	$-c < x < c$	
• $\|x\| \leq c$	$-c \leq x \leq c$	
• $\|x\| > c$	$x < -c$ or $c < x$	
• $\|x\| \geq c$	$x \leq -c$ or $c \leq x$	

Example 1: Solve the absolute value inequality

$$|x - 2| + 2 < 3$$

Example 2: Solve the inequality

$$|x + 1| \geq 1$$

Example 3: Solve the absolute value inequality. Express the answer using interval notation and graph the solution set.

- $3 - |2x + 4| \leq 1$

- $\left|\frac{2x-3}{2} + \frac{1}{3}\right| > 1$