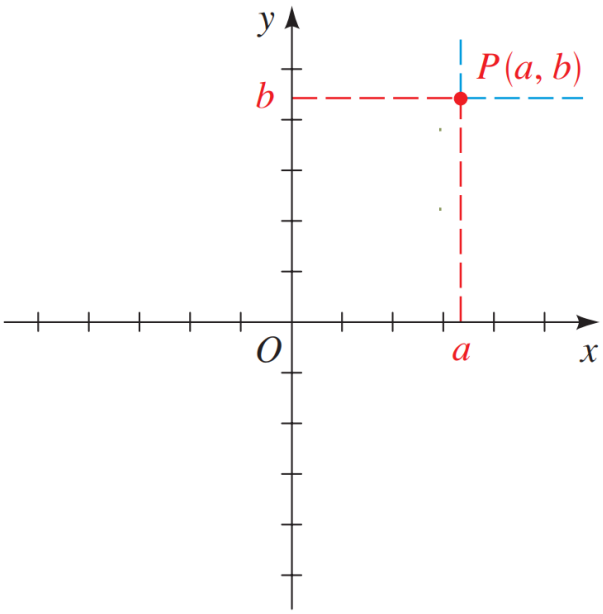
The Coordinate Plane

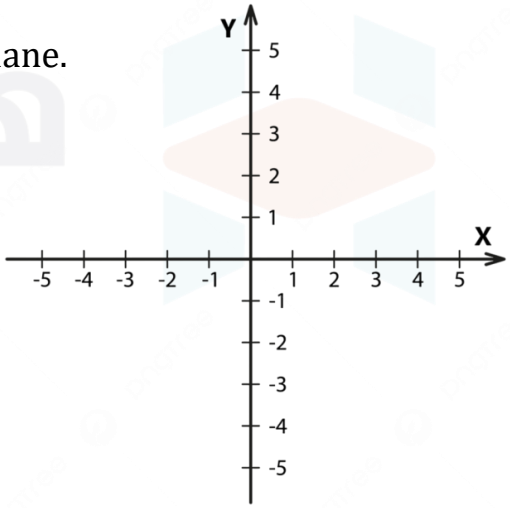


$a \Rightarrow x$-coordinate

$b \Rightarrow y$-coordinate

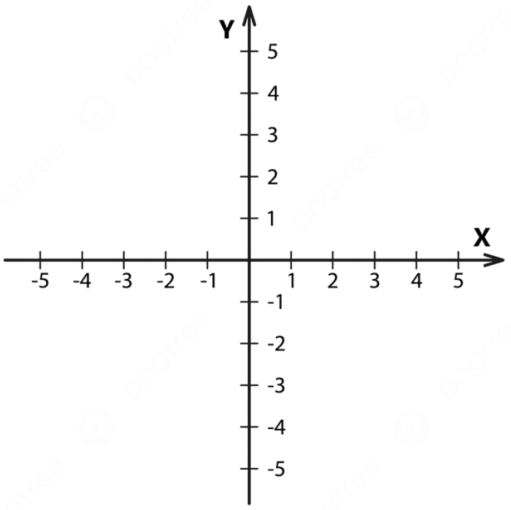
Example 1: Plot the given points in a coordinate plane.

$(0, 5), (-1, 0)$



Example 2: Sketch the region given by the set.

$\{(x, y) \mid x \leq 2\}$



The Distance and Midpoint Formulas

Distance Formula

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in is

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Horizontal distance: $d(A,B) = |x_2 - x_1|$

Vertical distance: $d(A,B) = |y_2 - y_1|$

Midpoint Formula

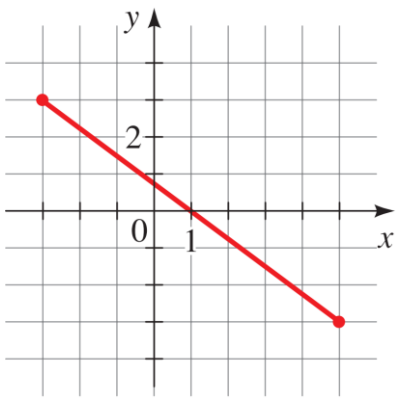
The midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 3: Find the distance between the points and Find the midpoint of the segment that joins them.

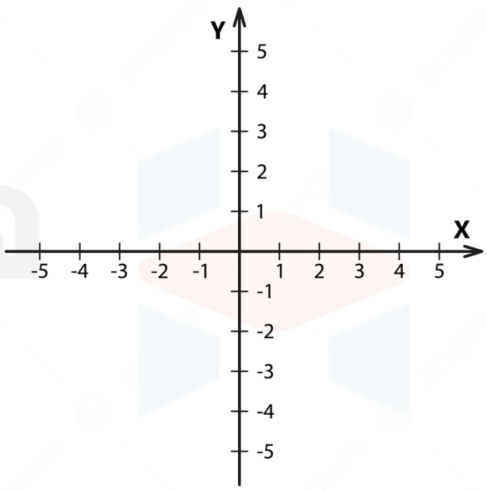
$$(16, -2), (-6, 2)$$

Example 4: A pair of points is graphed. (a) Find the distance between them. (b) Find the midpoint of the segment that joins them.

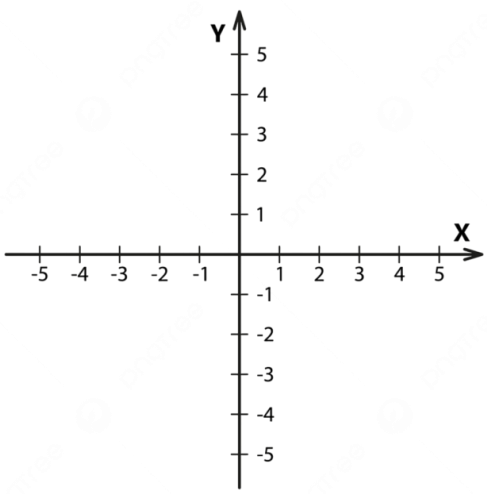


Example 5: Which of the points $A(6,7)$ or $B(-5,8)$ is closer to the origin?

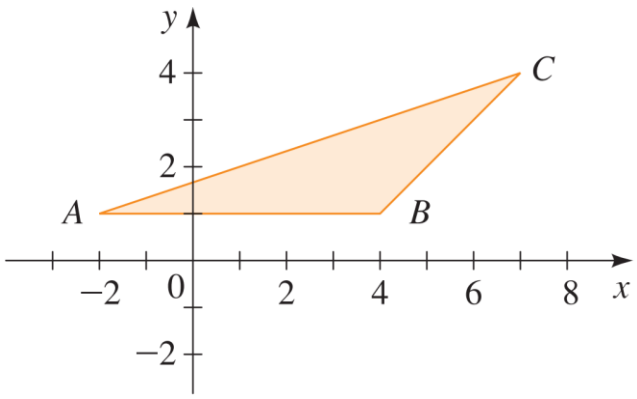
Example 6: Draw the rectangle with vertices $A(1,3), B(5,3), C(1,-3)$, and $D(5,-3)$ on a coordinate plane. Find the area of the rectangle.



Example 7: Draw the parallelogram with vertices $A(1,2), B(5,2), C(3,6)$, and $D(7,6)$ on a coordinate plane. Find the area of the parallelogram.



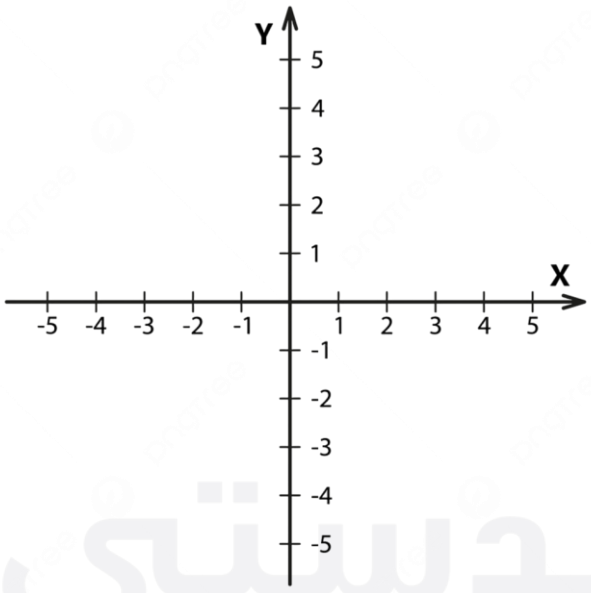
Example 8: Find the area of the triangle shown in the figure.



Example 9: If $M(6,8)$ is the midpoint of the line segment AB and if A has coordinates $(2,3)$, find the coordinates of B.

Graphs of Equations in Two Variables

Example 1: Sketch the graph of the equation $x - y = 2$



x	y	(x, y)

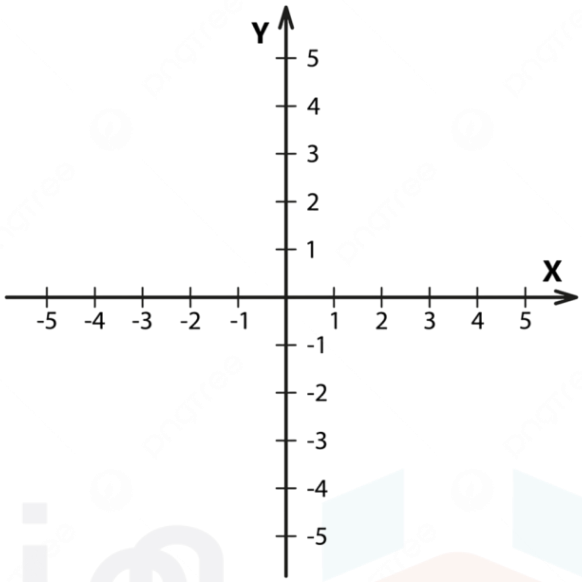
Example 2: Determine whether the given points are on the graph of the equation.

$$x^2 + xy + y^2 = 4; \quad (0, -2), (1, -2), (2, -2)$$

Example 3: Make a table of values and sketch the graph of the equation.

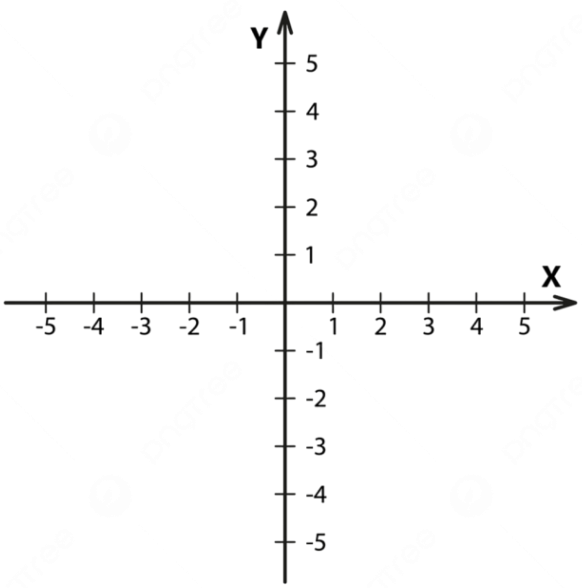
- $x^2 - y = 2$

x	y	(x, y)



- $y = |x| - 1$

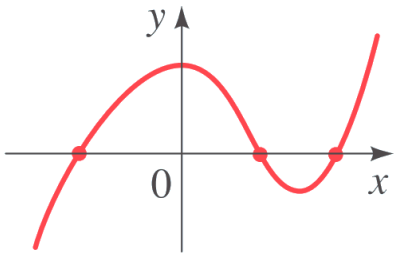
x	y	(x, y)



Intercepts

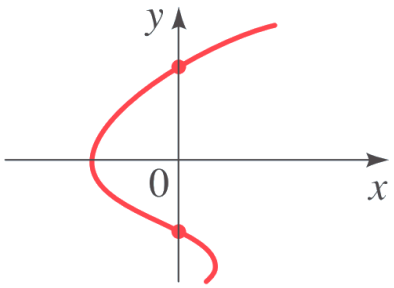
To find x-intercepts:

$$Set\ y\ =\ 0\ and\ solve\ for\ x$$



To find y-intercepts:

$$Set\ x\ =\ 0\ and\ solve\ for\ y$$

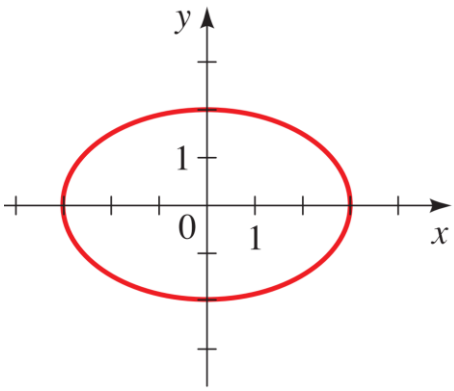


Example 1: Find the x-and y-intercepts of the graph of the equation.

- $y = x^2 - 5$
- $x^2 - xy + 3y = 1$
- $9x^2 - 4y^2 = 36$
- $y = \sqrt{x^2 - 16}$

Example 2: Find the x- and y-intercepts.

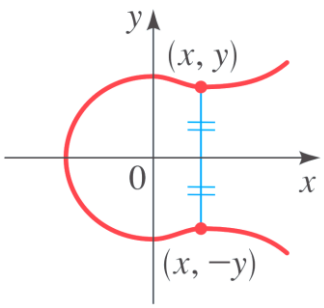
- $4x^2 + 9y^2 = 36$



- $x^4 + y^2 - xy = 16$

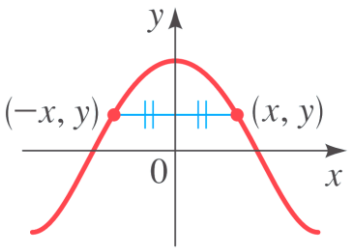
Symmetry

Types of Symmetry



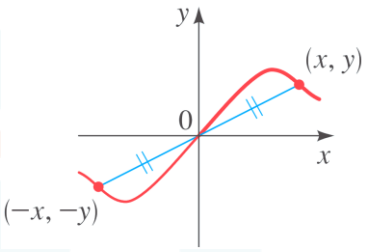
With respect to the x-axis (Mirror about x)

No Change in the equation if we replace y by $-y$.



With respect to the y-axis (Mirror about y)

No Change in the equation if we replace x by $-x$.



With respect to the $Origin$ (Mirror about origin)

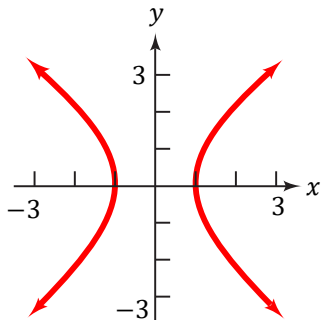
No Change in the equation if we replace x by $-x$ and y by $-y$.

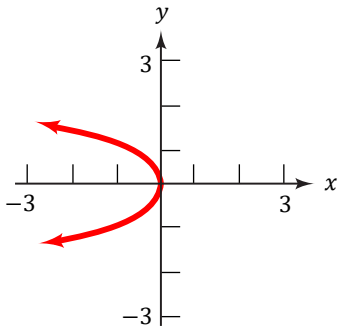
Example 1: Test the equation for symmetry.

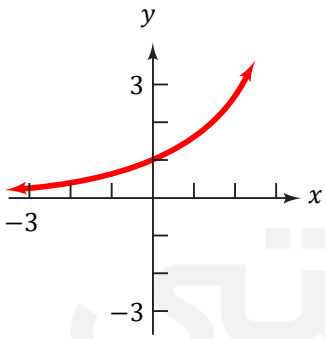
- $y = x^4 + x^2$

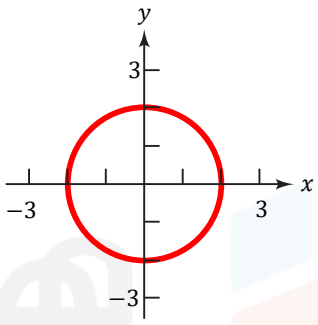
- $y = x^2 + |x|$

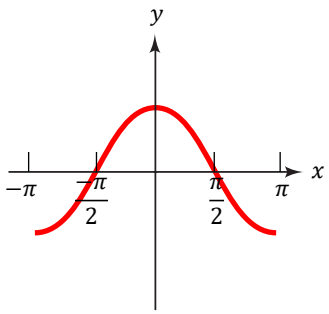
Example 2: What is the type of symmetry?

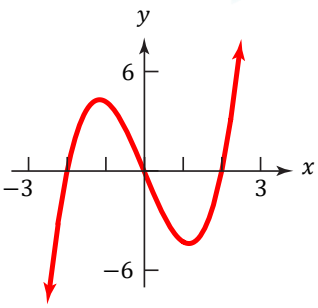


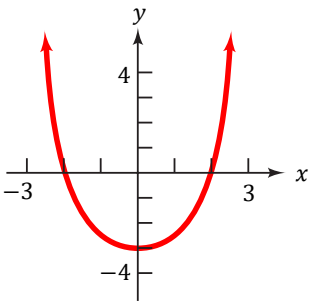






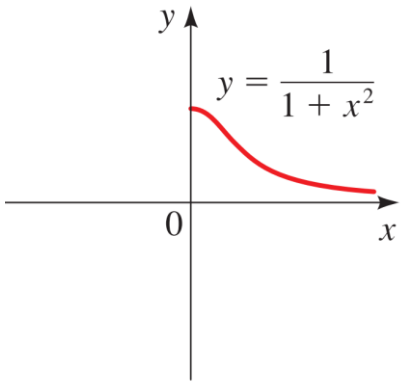




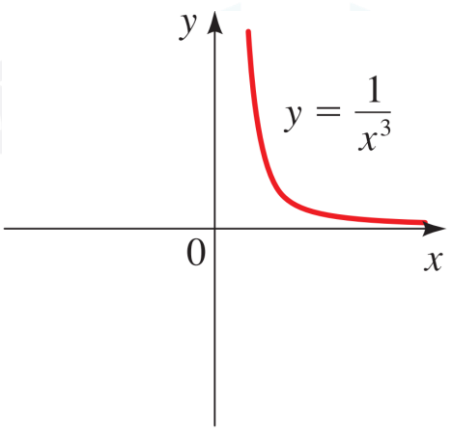


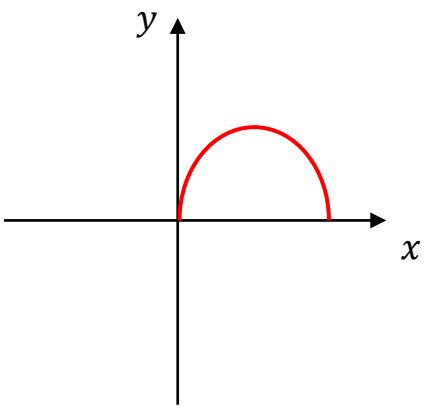
Example 3: Complete the graph using the given symmetry property.

- Symmetric with respect to the y-axis



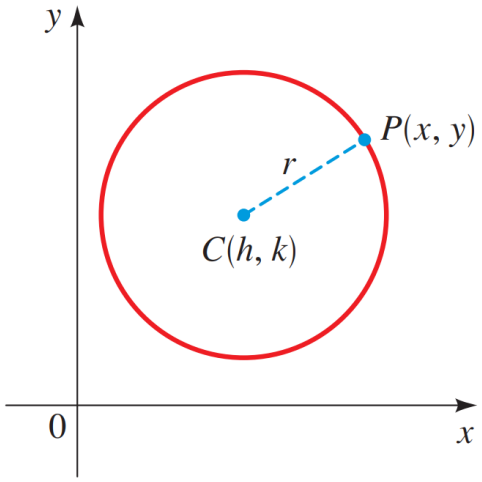
- Symmetric with respect to the origin





Circles

Equation of a Circle



An equation of the circle with center (h,k) and radius r is

$(x-h)^2+(y-k)^2=r^2$ (**the standard form**)

- If the center of the circle is the origin $(0,0)$ then the equation is

$$x^2+y^2=r^2$$

- If the equation of a circle is in **General Form**

$$x^2+y^2+ax+by+c=0$$

⇒ Complete the Squares in **x** and **y** to get the standard form

Example 1: Find an equation of the circle that satisfies the given conditions.

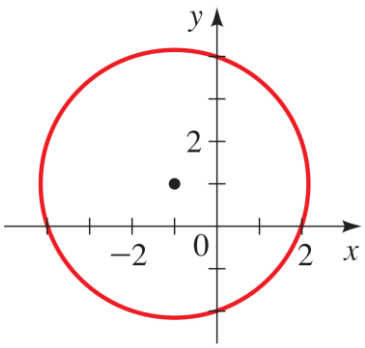
(a) Center $(2,-1)$; radius 3

(b) Endpoints of a diameter are $P(-1,1)$ and $Q(5,9)$

Example 2: Find the center and radius of the circle.

- $x^2 + y^2 = 9$
- $(x + 1)^2 + (y + 2)^2 = 36$

Example 3: Find the equation of the circle shown in the figure.



Example 4: Show that the equation represents a circle and find the center and radius of the circle.

- $2x^2 + 2y^2 - 3x = 0$
- $x^2 + y^2 + 4x - 6y + 12 = 0$