



Classify Numbers

Natural numbers: 1,2,3,4, ...

Integers: ..., -3, -2, -1,0,1,2,3,4, ...

Rational numbers: ratio of integers

$$r = \frac{m}{n} \quad \frac{1}{2} \quad -\frac{3}{7} \quad 46 = \frac{46}{1}$$

$$\frac{2}{3} = 0.66666 \dots = 0.\bar{6} \quad \frac{1}{2} = 0.5000 \dots = 0.5\bar{0}$$

$$\frac{9}{7} = 1.285714285714 \dots = 1.\overline{285714}$$

Irrational numbers:

$$\sqrt{3} \quad \sqrt{5} \quad \sqrt[3]{2} \quad \pi \quad \frac{3}{\pi^2}$$

Exercises

List the given elements to (a) natural numbers (b) integers (c) rational numbers (d) irrational numbers.

- $\left\{-1.5, 0, \frac{5}{2}, \sqrt{7}, 2.71, -\pi, 3.1\bar{4}, 100, -8\right\}$

- $\left\{1.3, 1.3333 \dots, \sqrt{5}, 5.34, -500, 1\frac{2}{3}, \sqrt{16}, \frac{246}{579}, -\frac{20}{5}\right\}$



Properties of Real Numbers

Commutative Properties

- $a + b = b + a$
- $ab = ba$

Associative Properties

- $a + (b + c) = (a + b) + c$
- $a(bc) = (ab)c$

Distributive Property

- $a(b + c) = ab + ac$
- $(b + c)a = ab + ac$

Exercises

If $2x+3x=(2+3)x$, the property is called Property.

State the property of real numbers being used.

- $3 + 7 = 7 + 3$
- $(x + 2y) + 3z = x + (2y + z)$
- $(5x + 1)3 = 15x + 3$
- $(x + a)(x + b) = (x + a)x + (x + a)b$

Rewrite the expression using the given property of real numbers.

- Commutative Property of Addition, $x + 3 =$
- Associative Property of Multiplication, $7(3x) =$
- Distributive Property, $4(A + B) =$



Addition and Subtraction

Properties of Negatives

- $(-1)a = -a$
- $-(-a) = a$
- $(-a)b = a(-b) = -(ab)$
- $(-a)(-b) = ab$
- $-(a + b) = -a - b$
- $-(a - b) = b - a$

Multiplication and Division

Properties of Fractions

- $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
- $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
- $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
- $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
- $\frac{ac}{bc} = \frac{a}{b}$
- if $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$





Exercises

Evaluate each expression

- $18 - 5 \cdot 2$

- $6 - [3 \cdot 5 + 2 \cdot (3 - 2)]$

- $\frac{4+8}{5-3}$

- $\frac{4}{3} + \frac{1}{2}$

- $\frac{2}{3} + \frac{4}{5} \cdot \frac{1}{6}$

- $\frac{5}{24} - \frac{8}{15}$

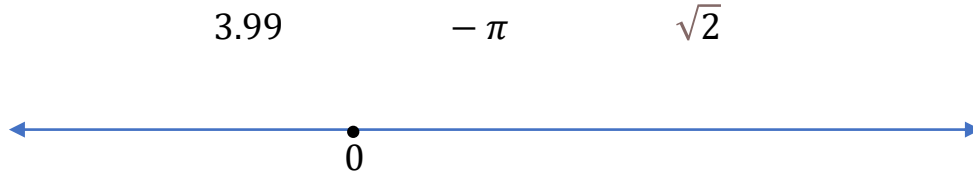
- $1 + \frac{5}{8} - \frac{1}{6}$

- $\frac{2}{2/3} - \frac{2/3}{2}$

- $\frac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}}$

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The Real Line



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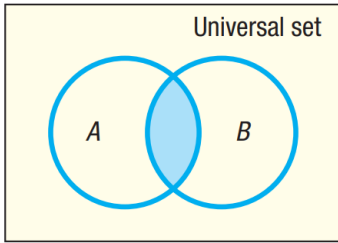
Sets

A set is a collection of objects, and these objects are called the elements of the set.

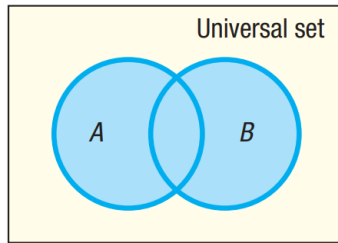
$$\text{Let } A = \{1, 2, 3, 4, 5, 6\}$$

A in *set notation*, $A = \{x \mid x \text{ is an integer and } 0 < x < 7\}$

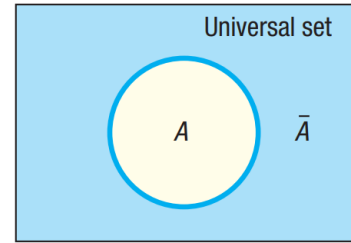
Union and Intersection of Set with Venn diagrams



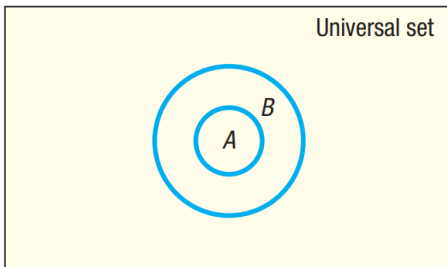
$A \cap B$
intersection



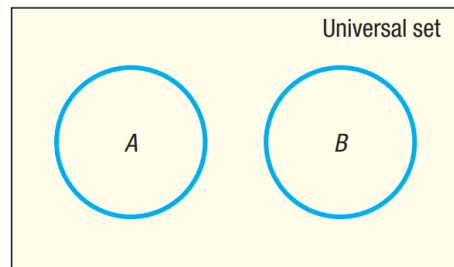
$A \cup B$
union



\bar{A}
complement



$A \subseteq B$
subset



$A \cap B = \emptyset$
disjoint sets

Example 1

If the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and if $A = \{1, 3, 5, 7, 9\}$, then $\bar{A} = \dots$



Example 2

If $S = \{1,2,3,4,5\}$, $T = \{4,5,6,7\}$, and $V = \{6,7,8\}$, find the sets $S \cup T$, $S \cap T$, and $S \cap V$.

Example 3

Use $U = \text{universal set} = \{0,1,2,3,4,5,6,7,8,9\}$, $A = \{1,3,4,5,9\}$, $B = \{2,4,6,7,8\}$, and $C = \{1,3,4,6\}$ to find the following:










- $A \cup B$
- $A \cap C$
- $(A \cup B) \cap C$
- $\overline{A \cap B}$



Intervals

$$(a, b) = \{x \mid a < x < b\}$$

$$[a, b] = \{x \mid a \leq x \leq b\}$$

Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	
$[a, b)$	$\{x \mid a \leq x < b\}$	
$(a, b]$	$\{x \mid a < x \leq b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \leq x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \leq b\}$	
$(-\infty, \infty)$	\mathbb{R} (set of all real numbers)	

Finding Unions and Intersections of Intervals

Example: Graph each set

$$(1,3) \cap [2,7]$$

$$(1,3) \cup [2,7]$$





Exercises

Find the indicated set if

$$A = \{x \mid x \geq -2\} \quad B = \{x \mid x < 4\} \quad C = \{x \mid -1 < x \leq 5\}$$

- $B \cup C$

- $A \cap B$

Express the inequality in interval notation, and then graph the corresponding interval.

- $x \leq 1$

- $1 \leq x \leq 2$

- $-2 < x \leq 1$

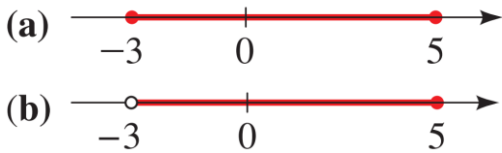
- $x > -1$



Express the interval in terms of inequalities, and then graph the interval.

- $(-3,0)$
- $(2,8]$

Express each set in interval notation.



Graph the set.

- $(-2,0) \cup (-1,1)$
- $[-4,6] \cap [0,8)$
- $(-\infty, -4) \cup (4, \infty)$





Absolute Value and Distance

If a is a real number, then the absolute value of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Absolute value properties

- $|a| \geq 0$
- $|a| = |-a|$
- $|ab| = |a||b|$
- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
- $|a + b| \leq |a| + |b|$

Distance between Points on the Real Line

If a and b are real numbers, then the **distance** between the points a and b on the real line is

$$d(a, b) = |b - a|$$

Example 1

The distance between the numbers -8 and 2 is

Example 2: Express the quantity without using absolute value

- $|a - b|$, where $a < b$
- $a + b + |a - b|$, where $a < b$



Example 3: Evaluate each expression

- $|100|$

- $|-73|$

- $|\sqrt{5} - 5|$

- $|10 - \pi|$

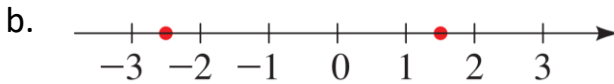
- $||-6| - |-4||$

- $\left| \frac{7-12}{12-7} \right|$

Example 4: Find the distance between the given numbers



d. -3 and 21



e. $\frac{11}{8}$ and $-\frac{3}{10}$

c. 2 and 17



Approximations

$$\sqrt{2} \approx 1.4142$$

$$\pi \approx 3.1416$$

Example:

Approximate 16.98752 to two decimal places by

(a) Truncating

(b) Rounding

Exercises

Approximate each number (a) rounded and (b) truncated to three decimal places.

- 18.9526
- 25.86134
- 28.65319
- 0.06291
- 1.0006
- $\frac{3}{7}$





Exponential Notation

If a is any real number and n is a positive integer, then the n th power of a is

$$a^n = \underbrace{a \cdot a \cdots a}_{n \text{ factors}}$$

The number a is called the base, and n is called the exponent.

Zero and Negative Exponents

If $a \neq 0$ is a real number and n is a positive integer, then

$$a^0 = 1, \quad a^{-n} = \frac{1}{a^n}$$

Law of Exponent

- $a^m a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$



Example 1:

Eliminate negative exponents and simplify each expression.

$$(a) \frac{6st^{-4}}{2s^{-2}t^2}$$

$$(b) \left(\frac{y}{3z^3}\right)^{-2}$$

$$\bullet a^m a^n = a^{m+n}$$

$$\bullet \frac{a^m}{a^n} = a^{m-n}$$

$$\bullet (a^m)^n = a^{mn}$$

$$\bullet (ab)^n = a^n b^n$$

$$\bullet \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\bullet \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\bullet \frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$$

Example 2: We can write the product $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as using exponential notation.

Example 3: Evaluate each expression.

$$\bullet -2^6$$

$$\bullet (-2)^6$$

$$\bullet \left(\frac{1}{5}\right)^2 \cdot (-3)^3$$



Example 4: Evaluate each expression.

$$\bullet \left(\frac{5}{3}\right)^0 \cdot 2^{-1}$$

$$\bullet -2^{-3} \cdot (-2)^0$$

$$\bullet \left(\frac{-2}{3}\right)^{-3}$$

$$\bullet 5^3 \cdot 5$$

$$\bullet 3^2 \cdot 3^0$$

$$\bullet (2^2)^3$$

$$\bullet \frac{10^7}{10^4}$$

Example 5: Simplify each expression.

$$\bullet x^2 x^3$$

$$\bullet (-x^2)^3$$

$$\bullet t^{-3} t^5$$

$$\bullet w^{-2} w^{-4} w^5$$

$$\bullet \frac{y^{10} y^0}{y^7}$$

$$\bullet (2x)^2 (5x^6)$$

$$\bullet (2a^3 a^2)^4$$



Simplify each expression and eliminate any negative exponent(s).

- $(2a^2b^{-1})(3a^{-2}b^2)$

- $(9y^{-2}z^2)(3y^3z)$

- $(8x^7y^2)\left(\frac{1}{2}x^3y\right)^{-2}$

- $\frac{x^2y^{-1}}{x^{-5}}$

- $\frac{3x^{-2}y^5}{9x^{-3}y^2}$

- $\left(\frac{y^{-1}}{x^{-2}}\right)^{-1} \left(\frac{3x^{-3}}{y^2}\right)^{-2}$

- $\frac{\frac{1}{2}a^{-3}b^{-4}}{2a^{-5}b^{-1}}$

- $\left(\frac{q^{-1}r^{-1}s^{-2}}{r^{-5}sq^{-8}}\right)^{-1}$

- $\left(\frac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3}$





Scientific Notation

A positive number x is said to be written in *scientific notation* if it is expressed as follows:

$$x = a \times 10^n \quad \text{where } 1 \leq a < 10 \text{ and } n \text{ is an integer}$$

$$3 \times 10^9 = 3000,000,000$$

$$1.2 \times 10^{-8} = 0.000000012$$

Example 1:

Write each number in scientific notation.

(a) 56920

(b) 0.000093

Example 2:

Write each number in scientific notation.

• 69,300,000

• 7,200,000,000,000

• 0.000028536

• 0.0001213



Example 3:

Write each number in decimal notation.

- 3.19×10^5

- 2.670×10^{-8}

- 7.1×10^{14}

- 8.55×10^{-3}

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