Simplifying Rational Expressions

$\frac{x^3-x}{x^2-5x+6}$ ⇒ Rational Expression, $\frac{x}{\sqrt{x^2+1}}$ ⇒ Not rational(Fractional)

Cancellation Property

$$\frac{ac}{bc}=\frac{a}{b} \quad (\text{if } b\neq 0, c\neq 0)$$

Example 1: Simplify

$$\frac{x^2-1}{x^2+x-2}$$

Example 2: Which of the following are rational expressions?

(a) $\frac{3x}{x^2-1}$

(b) $\frac{\sqrt{x+1}}{2x+3}$

(c) $\frac{x(x^2-1)}{x+3}$

Example 3: Reduce each rational expression to lowest terms.

- $\frac{3x+9}{x^2-9}$
- $\frac{5(x-3)(2x+1)}{10(x-3)^2}$

- $\frac{3y^2-y-2}{3y^2+5y+2}$

- $\frac{1-x^2}{x^3-1}$

Multiplying and Dividing Rational Expressions

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational expressions, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{if } b \neq 0, d \neq 0$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \quad \text{if } b \neq 0, c \neq 0, d \neq 0$$

Example 1: Perform the indicated multiplication and simplify:

$$\frac{x^2 + 2x - 3}{x^2 + 8x + 16} \cdot \frac{3x + 12}{x - 1}$$

Example 2: Perform the indicated division and simplify:

$$\frac{x - 4}{x^2 - 4} \div \frac{x^2 - 3x - 4}{x^2 + 5x + 6}$$

Example 3: Perform the multiplication or division and simplify.

- $\frac{3x+6}{5x^2} \cdot \frac{x}{x^2-4}$
- $\frac{t-3}{t^2+9} \cdot \frac{t+3}{t^2-9}$
- $\frac{\frac{5x^2-7x-6}{2x^2+3x-5}}{\frac{15x^2+14x+3}{2x^2+13x+20}}$
- $\frac{x^2+2x-3}{x^2-2x-3} \cdot \frac{3-x}{3+x}$
- $\frac{2x+1}{2x^2+x-15} \div \frac{6x^2-x-2}{x+3}$
- $\frac{x^2+x-6}{x^2+4x-5} \cdot \frac{x^2-25}{x^2+2x-15}$

Adding and Subtracting Rational Expressions with Equal Denominators

If $\frac{a}{b}$ and $\frac{c}{b}$ are two rational expressions, then

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \qquad \frac{a}{b}-\frac{c}{b}=\frac{a-c}{b} \quad \text{if } b \neq 0$$

Example 1: Perform the indicated operations and simplify.

$$\frac{2x^2-4}{2x+5}+\frac{x+3}{2x+5} \qquad x \neq -\frac{5}{2}$$

Example 2: Perform the indicated operations and simplify.

$$\frac{2x}{x-3}+\frac{5}{3-x} \qquad x \neq 3$$

Adding and Subtracting Rational Expressions with Unequal Denominators

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational expressions, then

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd} \quad \text{if } b\neq 0, d\neq 0$$

$$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd} \quad \text{if } b\neq 0, d\neq 0$$

Example 3: Perform the indicated operations and simplify.

$$\frac{3}{x-1}+\frac{x}{x+2}$$

Example 4: Choose the statement that is not true. Assume $b\neq 0, c\neq 0$, and $d\neq 0$ as necessary.

(a) $\frac{ac}{bc}=\frac{a}{b}$

(b) $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$

(c) $\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$

(d) $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{ac}{bd}$

Example 5: Perform the addition or subtraction and simplify

- $\frac{x}{2}+\frac{5}{2}$
- $\frac{x}{x-4}-\frac{3}{x+6}$
- $1+\frac{1}{x+3}$
- $\frac{1}{x^2}+\frac{1}{x^2+x}$
- $\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}$
- $\frac{2}{x}+\frac{3}{x-1}-\frac{4}{x^2-x}$

- $\frac{1}{x^2+3x+2} - \frac{1}{x^2-2x-3}$

Compound (Complex) Fractions

Compound fraction:
Numerator or denominator, or both are fractional expressions

Example 1: Simplify:

$$\frac{\frac{x}{y}+1}{1-\frac{y}{x}}$$

Example 2: Simplify the compound fractional expression.

- $\frac{1+\frac{1}{x}}{\frac{1}{x}-2}$
- $\frac{\frac{x-3}{x-4}-\frac{x+2}{x+1}}{x+3}$

- $x-\frac{y}{\frac{x}{y}+\frac{y}{x}}$
- $\frac{x^{-1}+y^{-1}}{(x+y)^{-1}}$
- $\frac{(1-x^2)^{1/2}+x^2(1-x^2)^{-1/2}}{1-x^2}$
- $\frac{4(x+2)^{-1}-3}{3(x+2)^{-1}-1}$

Avoiding Common Errors

Correct multiplication property	Common error with addition
$(a \cdot b)^2 = a^2 \cdot b^2$	$(a + b)^2 = a^2 + b^2$
$\sqrt{a \cdot b} = \sqrt{a}\sqrt{b}\ (a, b \geq 0)$	$\sqrt{a + b} = \sqrt{a} + \sqrt{b}$
$\sqrt{a^2 \cdot b^2} = a \cdot b\ (a, b \geq 0)$	$\sqrt{a^2 + b^2} = a + b$
$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$	$\frac{1}{a} + \frac{1}{b} = \frac{1}{a + b}$
$\frac{ab}{a} = b$	$\frac{a + b}{a} = b$
$a^{-1} \cdot b^{-1} = (a \cdot b)^{-1}$	$a^{-1} + b^{-1} = (a + b)^{-1}$