Solving Linear Equations

$x + 4 = 7 \quad \Rightarrow$ Equation

$x \Rightarrow unknown\ (variable)$
$values\ of\ x \Rightarrow solutions\ (roots)$
$finding\ the\ solution \Rightarrow solving\ the\ equation$

Properties of equality

- $A = B \Leftrightarrow A + C = B + C$

- $A = B \Leftrightarrow CA = CB\ (C \neq 0)$

Solving Linear Equations

A linear equation in one variable is

$$\boldsymbol{ax + b = 0}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real numbers and $\boldsymbol{x}$ is the variable.

Linear equations

$4x - 5 = 3$

$2x = \frac{1}{2}x - 7$

$x - 6 = \frac{x}{3}$

Nonlinear equations

$x^2 + 2x = 8$

$\sqrt{x} - 6x = 0$

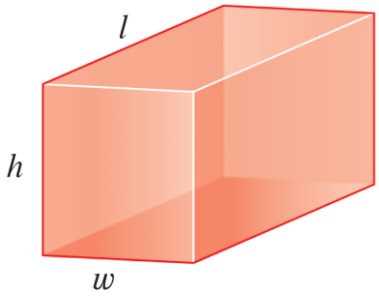
$\frac{3}{x} - 2x = 1$

Example 1: solve the equation

$$7x - 4 = 3x + 8$$

Solving for One Variable in Terms of Others

Example 2: The surface area A of the closed rectangular box can be calculated from the length l, the width w, and the height h according to the formula



$$A = 2lw + 2wh + 2lh$$

Solve for w in terms of the other variables in this equation.

Example 3: Determine whether the given value is a solution of the equation.

- $4x + 7 = 9x - 3$
  (a) $x = -2$
  (b) $x = 2$

- $\frac{1}{x} - \frac{1}{x-4} = 1$
  (a) $x = 2$,
  (b) $x = 4$

Example 4: The given equation is either linear or equivalent to a linear equation. Solve the equation.

- $2x + 3 = 7 - 3x$
- $2(1 - x) = 3(1 + 2x) + 5$
- $\frac{4}{x-1} + \frac{2}{x+1} = \frac{35}{x^2-1}$
- $(t - 4)^2 = (t + 4)^2 + 32$

Example 5: Solve the equation for the indicated variable.

- $PV = nRT$; for R
- $\frac{a+1}{b} = \frac{a-1}{b} + \frac{b+1}{a}$; for a
- $\frac{ax+b}{cx+d} = 2$; for x

Solving Quadratic Equations

Quadratic Equations

A quadratic equation is

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers with $a \neq 0$.

Zero – Product Property

$$(A)(B) = 0 \text{ if and only if } A = 0 \text{ or } B = 0$$

Solving a Quadratic Equation by Factoring

Example 1: Find all real solutions of the equation $x^2 + 5x = 24$.

Solving a simple quadratic Equation

The solutions of $x^2 = c \Rightarrow x = \sqrt{c}$ and $x = -\sqrt{c}$.

Example 2: Find all real solutions of each equation.

(a) $x^2 = 5$

(b) $(x - 4)^2 = 5$

Completing the Square

To make $x^2 + bx$ a perfect square, add $\left(\frac{b}{2}\right)^2$, (**the square of half the coefficient of x**). This gives the perfect square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example 3: Find all real solutions of each equation.

(a) $x^2 - 8x + 13 = 0$

(b) $3x^2 - 12x + 6 = 0$

The quadratic formula

The roots of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 4: Find all real solutions of the equation.

$$4x^2 + 12x + 9 = 0$$

The Discriminant

The **discriminant** of the general quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$ is $D = b^2 - 4ac$.

- If $D > 0$, then the equation has two distinct real solutions.
- If $D = 0$, then the equation has exactly one real solution.
- If $D < 0$, then the equation has no real solution.

Example 5: Use the discriminant to determine how many real solutions each equation has.

(a) $x^2 + 4x - 1 = 0$

(b) $4x^2 - 12x + 9 = 0$

(c) $\frac{1}{3}x^2 - 2x + 4 = 0$

Example 6: Find all real solutions of the equation by factoring

- $x^2 + x - 12 = 0$
- $2y^2 + 7y + 3 = 0$
- $(2x - 5)^2 = 81$

Example 7: Find all real solutions of the equation by completing the square.

- $x^2 + 2x - 5 = 0$
- $3x^2 - 6x - 1 = 0$

Example 8: Find all real solutions of the quadratic equation using quadratic formula.

- $x^2 - 2x - 15 = 0$
- $9x^2 + 12x + 4 = 0$

Example 9: Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

- $x^2 - 6x + 1 = 0$
- $4x^2 + 5x + \frac{13}{8} = 0$

Solving other Types of Equations

An Equation Involving Fractional Expressions

Example 1: Solve the equation

$$\frac{3}{x}-\frac{2}{x-3}=\frac{-12}{x^2-9}$$

An Equation Involving a Radical

Example 2: Solve the equation

$$2x=1-\sqrt{2-x}$$

A Fourth-Degree Equation of Quadratic Type

Example 3: Find all solutions of the equation

$$x^4 - 8x^2 + 8 = 0$$

An Equation Involving Fractional Powers

Example 4: Find all solutions of the equation

$$x^{1/3} + x^{1/6} - 2 = 0$$

An Absolute Value Equation

$|X| = C$ is equivalent to $X = C$ or $X = -C$

Example 5: Solve the equation

$$|2x - 5| = 3$$

Example 6: Find all real solutions of the equation

- $\frac{x^2}{x+100} = 50$
- $\sqrt{2x-1} = \sqrt{3x-5}$
- $\frac{x}{2x+7} - \frac{x+1}{x+3} = 1$
- $x^4 - 5x^2 + 4 = 0$
- $|3x+5| = 1$
- $|x-6| = -1$

- $\sqrt{x} - 3\sqrt[4]{x} - 4 = 0$

- $4(x+1)^{\frac{1}{2}} - 5(x+1)^{\frac{3}{2}} + (x+1)^{\frac{5}{2}} = 0$