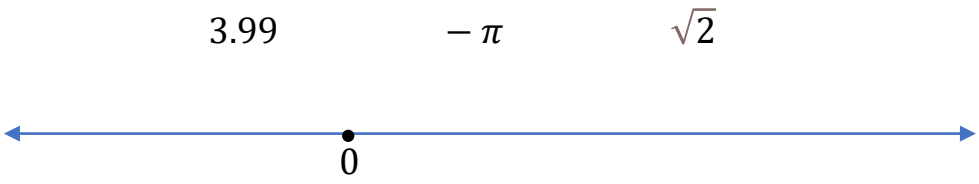
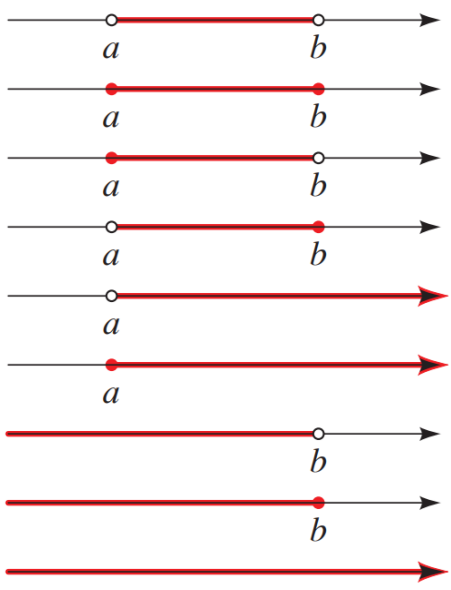
The Real Line



Intervals

$$(a, b) = \{x \mid a < x < b\} \qquad [a, b] = \{x \mid a \le x \le b\}$$

Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \le x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

Finding Unions and Intersections of Intervals

Example: Graph each set

(1,3) ∩ [2,7]

(1,3) ∪ [2,7]

Exercises

Find the indicated set if

$$A = \{x \mid x \geq -2\} \qquad B = \{x \mid x < 4\} \qquad C = \{x \mid -1 < x \leq 5\}$$

- $B \cup C$

- $A \cap B$

Express the inequality in interval notation, and then graph the corresponding interval.

- $x \leq 1$

- $1 \leq x \leq 2$

- $-2 < x \leq 1$

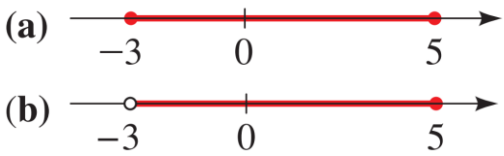
- $x > -1$

Express the interval in terms of inequalities, and then graph the interval.

- $(-3,0)$

- $(2,8]$

Express each set in interval notation.



Graph the set.

- $(-2,0) \cup (-1,1)$

- $[-4,6] \cap [0,8)$

- $(-\infty,-4) \cup (4,\infty)$

Absolute Value and Distance

If a is a real number, then the absolute value of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Absolute value properties

- $|a| \geq 0$
- $|a| = |-a|$
- $|ab| = |a||b|$
- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
- $|a+b| \leq |a| + |b|$

Distance between Points on the Real Line

If a and b are real numbers, then the **distance** between the points a and b on the real line is

$$d(a,b) = |b-a|$$

Example 1

The distance between the numbers -8 and 2 is .....

Example 2:. Express the quantity without using absolute value

- $|a-b|$, where $a < b$

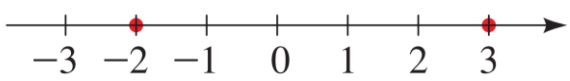
- $a+b+|a-b|$, where $a < b$

Example 3: Evaluate each expression

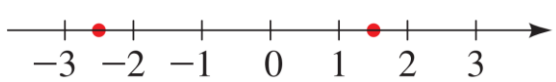
- $|100|$
- $|-73|$
- $|\sqrt{5}-5|$
- $|10-\pi|$
- $||-6|-|-4||$
- $\left|\frac{7-12}{12-7}\right|$

Example 4: Find the distance between the given numbers

a.



b.



c. 2 and 17

d. – 3 and 21

e. $\frac{11}{8}$ and $-\frac{3}{10}$

Exponential Notation

If a is any real number and n is a positive integer, then the n th power of a is

$$a^n = \underbrace{a.a\ldots.a}_{n\ factors}$$

The number a is called the base, and n is called the exponent.

Zero and Negative Exponents

If $a \neq 0$ is a real number and n is a positive integer, then

$$a^0 = 1, \qquad a^{-n} = \frac{1}{a^n}$$

Law of Exponent

- $a^m a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

Example 1:
Eliminate negative exponents and simplify each expression.

(a) $\frac{6st^{-4}}{2s^{-2}t^{2}}$

(b) $\left(\frac{y}{3z^{3}}\right)^{-2}$

- $a^m a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

Example 2: We can write the product 5 . 5 . 5 . 5 . 5 . 5 as ..... using exponential notation.

Example 3: Evaluate each expression.

- -2^6
- $(-2)^6$
- $\left(\frac{1}{5}\right)^2 \cdot (-3)^3$

Example 4: Evaluate each expression.

- $\left(\frac{5}{3}\right)^0 \cdot 2^{-1}$
- $-2^{-3} \cdot (-2)^0$
- $\left(\frac{-2}{3}\right)^{-3}$
- $5^3 \cdot 5$
- $3^2 \cdot 3^0$
- $(2^2)^3$
- $\frac{10^7}{10^4}$

Example 5: Simplify each expression.

- x^2x^3
- $(-x^2)^3$
- $t^{-3}t^5$
- $w^{-2}w^{-4}w^5$
- $\frac{y^{10}y^0}{y^7}$
- $(2x)^2(5x^6)$
- $(2a^3a^2)^4$

Simplify each expression and eliminate any negative exponent(s).

- $(2a^2b^{-1})(3a^{-2}b^2)$
- $(9y^{-2}z^2)(3y^3z)$
- $(8x^7y^2)\left(\frac{1}{2}x^3y\right)^{-2}$
- $\frac{x^2y^{-1}}{x^{-5}}$
- $\frac{3x^{-2}y^5}{9x^{-3}y^2}$
- $\left(\frac{y^{-1}}{x^{-2}}\right)^{-1}\left(\frac{3x^{-3}}{y^2}\right)^{-2}$
- $\frac{\frac{1}{2}a^{-3}b^{-4}}{2a^{-5}b^{-1}}$
- $\left(\frac{q^{-1}r^{-1}s^{-2}}{r^{-5}sq^{-8}}\right)^{-1}$
- $\left(\frac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3}$

Scientific Notation

A positive number $\boldsymbol{x}$ is said to be written in ***scientific notation*** if it is expressed as follows:

$$x = a \times 10^n \qquad where\ 1 \leq a < 10\ \ and\ \boldsymbol{n}\ is\ an\ integer$$

$3 \times 10^9 = 3000{,}000{,}000$ $\qquad$ $1.2 \times 10^{-8} = 0.000000012$

Example1:
Write each number in scientific notation.

(a) 56920 $\qquad$ (b) 0.000093

Example 2:

Write each number in scientific notation.

- 69,300,000

- 7,200,000,000,000

- 0.000028536

- 0.0001213

Example 3:

Write each number in decimal notation.

- 3.19×10^{5}

- 2.670×10^{-8}

- 7.1×10^{14}

- 8.55×10^{-3}