Radicals

Definition of n^th Root

If n is any positive integer, then the principal n^{th} root of a is defined as

$$\sqrt[n]{a} = b \quad means \qquad b^n = a$$

If n is even, $a \geq 0$ and $b \geq 0$

Properties of n^{th} Roots

- $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
- $\sqrt[n]{a^n} = a$ if n is odd
- $\sqrt[n]{a^n} = |a|$ if n is even

Example 1: Simplify the following Radicals

- $\sqrt{32}$
- $\sqrt[3]{-16x^4}$
- $-8\sqrt{12} + \sqrt{3}$
- $\sqrt[3]{8x^4} + \sqrt[3]{-x} + 4\sqrt[3]{27x}$

Example 2: Evaluate each expression

- $\sqrt{16}$
- $\sqrt[4]{16}$
- $\sqrt[3]{-64}$
- $\sqrt{\frac{27}{4}}$
- $\sqrt{7}\sqrt{28}$
- $\sqrt[3]{2}\sqrt[3]{32}$
- $\sqrt[6]{\frac{1}{2}}\sqrt[6]{128}$

Example 3: Simplify each expression. Assume that all variables are positive when they appear

- $\sqrt[3]{27}$
- $\sqrt[3]{-8}$
- $\sqrt{700}$
- $\sqrt[3]{-8x^4}$
- $\sqrt[4]{x^{12}y^8}$
- $\sqrt[3]{\sqrt{64x^6}}$

Example 4: Simplify each expression. Assume that all variables are positive when they appear

- $\sqrt[3]{\frac{3xy^2}{81x^4y^2}}$
- $\sqrt{15x^2}\sqrt{5x}$
- $(\sqrt[3]{3}\sqrt{10})^4$
- $(5\sqrt{8})(-3\sqrt{3})$
- $6\sqrt{5} - 4\sqrt{5}$

Example 5: Simplify the expression. Assume that all letters denote positive numbers

- $\sqrt{32} + \sqrt{18}$
- $\sqrt[3]{54} - \sqrt[3]{16}$
- $\sqrt{9a^3} - \sqrt{a}$
- $\sqrt[3]{2y^4} - \sqrt[3]{2y}$
- $\sqrt{81x^2 + 81}$

Rationalizing the Denominator; Standard Form

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \cdot 1 = \frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

$$\sqrt[n]{a^m}\sqrt[n]{a^{n-m}} = \sqrt[n]{a^{m+n-m}} = \sqrt[n]{a^n} = a$$

Note: **standard form = denominator with no radicals**

If a Denominator Contains the Factor	Multiply by	To Obtain a Denominator Free of Radicals
$\sqrt{3}$	$\sqrt{3}$	$(\sqrt{3})^2 = 3$
$\sqrt{3}+1$	$\sqrt{3}-1$	$(\sqrt{3})^2 - 1^2 = 3 - 1 = 2$
$\sqrt{2}-3$	$\sqrt{2}+3$	$(\sqrt{2})^2 - 3^2 = 2 - 9 = -7$
$\sqrt{5}-\sqrt{3}$	$\sqrt{5}+\sqrt{3}$	$(\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$
$\sqrt[3]{4}$	$\sqrt[3]{2}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$

Example 1:

Put each fractional expression into standard form by rationalizing the denominator.

(a) $\frac{2}{\sqrt{3}}$

(b) $\frac{1}{\sqrt[3]{5}}$

(c) $\sqrt[7]{\frac{1}{a^2}}$

Example 2: Rationalize the denominator of each expression. Assume that all variables are positive when they appear.

- $\frac{1}{\sqrt{2}}$
- $\frac{-\sqrt{3}}{\sqrt{5}}$
- $\frac{9}{\sqrt[4]{2}}$
- $\sqrt{\frac{x}{5}}$
- $\frac{1}{\sqrt[4]{x^3}}$
- $\frac{\sqrt{3}}{5-\sqrt{2}}$
- $\frac{2-\sqrt{5}}{2+3\sqrt{5}}$
- $\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

Rational Exponent

Definition of Rational Exponent

If m and n are integers and $n > 0$, then

$$a^{m/n} = (\sqrt[n]{a})^m \qquad \text{or} \qquad a^{m/n} = \sqrt[n]{a^m}$$

If n is even, then we require that $a \geq 0$.

Example 1:
Use the Laws of Exponents with Rational Exponents to simplify the following:

- $a^{1/3}a^{7/3}$
- $\dfrac{a^{2/5}a^{7/5}}{a^{3/5}}$

Example 2: complete the following table

Radical expression	Exponential expression
$\dfrac{1}{\sqrt{3}}$	
$\sqrt[3]{7^2}$	
	$4^{2/3}$
	$10^{-3/2}$
$\sqrt[5]{5^3}$	
	$2^{-1.5}$
	$a^{2/5}$

Example 3: Simplify the expression and eliminate any negative exponent(s). Assume that all letters denote positive numbers.

- $x^{3/4}x^{5/4}$
- $\frac{\left(2y^{4/3}\right)^{2}y^{-2/3}}{y^{7/3}}$
- $\left(8a^{6}b^{3/2}\right)^{2/3}$
- $(u^{4}v^{6})^{-1/3}$
- $\left(\frac{x^{-2/3}}{y^{1/2}}\right)\left(\frac{x^{-2}}{y^{-3}}\right)^{1/6}$
- $\left(\frac{x^{8}y^{-4}}{16y^{4/3}}\right)^{-1/4}$