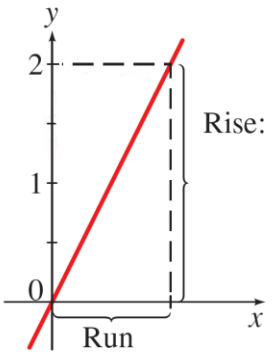
Lines

The Slope of a Line

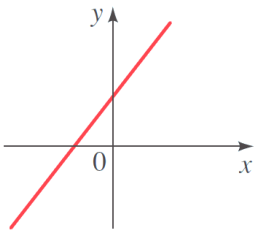
The slope m of a nonvertical line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

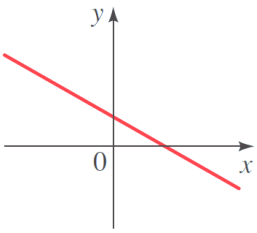


The slope of a vertical line is not defined.

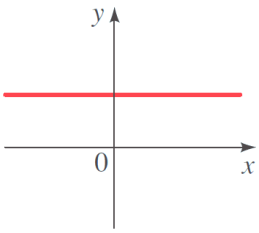
Different cases for the slope



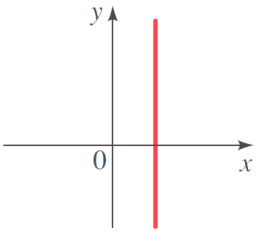
Positive slope
m> 0



Negative slope
m<0



Zero slope
m=0



No slope
m is undefined

Example 1: Find the slope of the line through P(-5, 1) and Q(3, – 2)

Point-Slope Form of the Equation of a Line

An equation of the line that passes through the point (x_1, y_1) and has slope m is

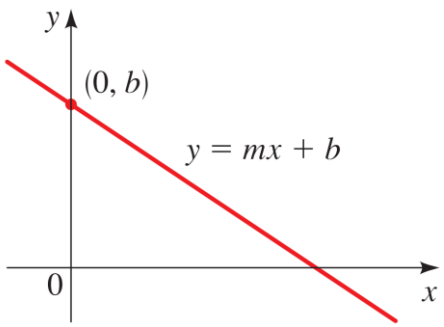
$$y - y_1 = m(x - x_1)$$

Example 2: Find an equation of the line through (2, 3) with slope 5

Slope-Intercept Form of the Equation of a Line

An equation of the line that has slope m and y-intercept b is

$$y = mx + b$$



Example 3: Find an equation of the line with slope $\frac{2}{5}$ and y-intercept 4 .

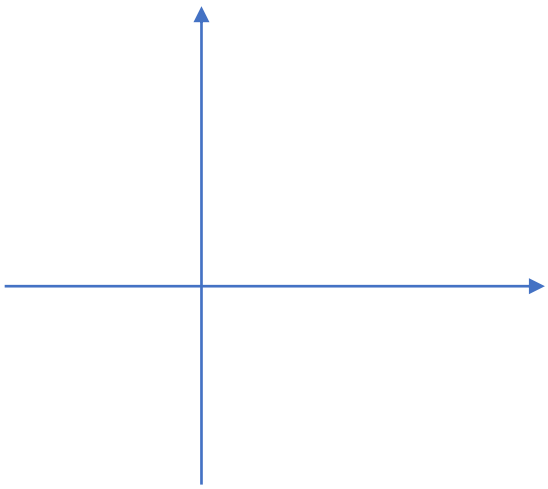
General Equation of a Line

$$Ax + By + C = 0 \quad \Rightarrow \quad \boldsymbol{y} = -\frac{A}{B}\boldsymbol{x} - \frac{C}{B} \qquad m = -\frac{A}{B}$$

$$\text{x – intercept} = -\frac{C}{A}$$

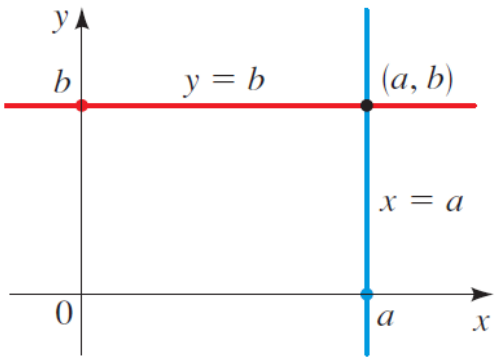
$$\text{y – intercept} = -\frac{C}{B}$$

Example 4: Sketch the graph of the equation $2x + 4y = 8$



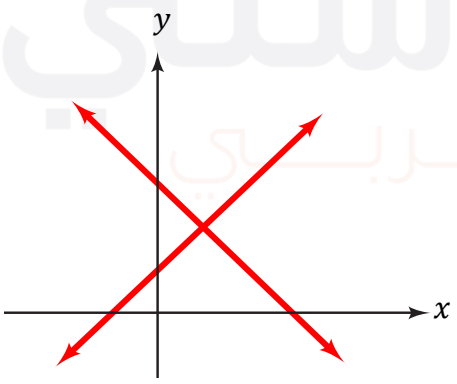
Vertical and Horizontal Lines

- An equation of the vertical line through (a, b) is $\boldsymbol{x = a}$.
- An equation of the horizontal line through (a, b) is $\boldsymbol{y = b}$.

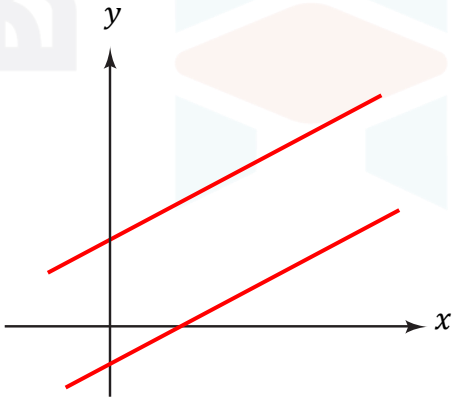


Example 5:
(a) Find an equation of the vertical line passes through the point $(1, -4)$
(b) Find an equation of the horizontal line passes through the point $(3, -1)$

Parallel and Perpendicular Lines



Perpendicular lines



Parallel lines

- Two nonvertical lines are parallel if and only if they have the same slope.
- Two lines with slopes m_1 and m_2 are perpendicular if and only if

$$m_1 m_2 = -1$$

- A horizontal line (slope=0) is perpendicular to a vertical line (no slope).

Example 6: Find the slope and y-intercept of the line

- $4x + 5y = 10$
- $3x - 4y = 12$
- $x = -5$
- $y = -2$

Example 7: Determine whether the lines are parallel, perpendicular, or neither

- $y = 2x + 3;\ \ 2y - 4x - 5 = 0$
- $7x - 3y = 2;\ 9y + 21x = 1$
- $6y - 2x = 5;\ 2y + 6x = 1$

Example 8: Find an equation of the line containing the points $(-2, 5)$ and $(-1, -3)$

Example 9: Find the x- and y-intercepts of the line.

$$6x - 7y - 42 = 0$$

Example 10: Find an equation of the line

(a) through $(1, 2)$ and parallel to the line $y = 3x - 5$

(b) through $(2, 6)$ and Perpendicular to the line $y = 1$